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Packet Radio Temporary Note #16

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Notes on Digital Modulation Techniques with Emphasis on Noise, Interference and Signal Capture Characteristics of FM(FSK) Systems

1.0 INTRODUCTION

The primary purpose of these notes is to discuss the signal to noise and signal to interference of FM systems with particular emphasis on capture effect of wide deviation FM systems. Some general comments relative to digital radio relay systems are included in Section 2 with the objective of clarifying some of the information to follow. The reader's attention is particularly called to the importance of the interference model discussed in Section 2.5.

It was thought desirable to compare FM (FSK) system with at least one other common digital data modulation technique and for this reason the subject of phase shift keyed systems is introduced briefly in Section 5. Some theoretical and measured characteristics are presented but no claim is made than an adequate comparison of the two techniques is made in these notes.

Though the writer is sure that these notes will leave many questions unanswered, some conclusions have been reached which should point the way to more detailed investigations. It is recognized that equipment economy is a major factor in the choice of optimum modulation technique for the ARPA packet transmission system, and this has influenced the direction of this investigation. Also, it has been shown that the interference model is a very key factor in defining the error rate threshold characteristics of both FM and FSK systems. On the assumption that a large number of interferors will be present, it is recommended that the worst-case interference be assumed to be modeled by Gaussian noise. It is shown that the high deviation ratio FM systems that are so effective in providing high signal-to-noise and "capture" of the desired signal in analog systems are not advantageous for digital systems because of the poor threshold characteristics of such systems. In

particular, it appears that if use is to be made of systems that expand bandwidth, a much more effective technique is to use coding techniques in conjunction with phase shift keying (PSK) techniques since optimum error rate performance can be obtained with PSK systems at low S/N and such systems have excellent "capture" characteristics. It would appear that further investigation should be directed toward economic coding techniques and economic PSK modulation techniques. Of particular importance in equipment design is the intermittent nature of the packet transmission technique which inhibits use of any system which will require significant signal acquisition time (compared to period of packet). However, this requirement does not appear to unduly restrict the potential use of PSK techniques.

In spite of the above comments it is recognized that the choice of modulation technique will be strongly influenced by conomics and this is favorable to FM. For this reason, FM (FSK) modulation might well be a baseline system against which other systems are evaluated.

2.0 SOME GENERAL CONSIDERATIONS

Most, if not all, digital transmission techniques exhibit a sharp threshold both as a function of signal-to-noise (S/N) in systems limited only by thermal noise and as a function of signal-to-interference (S/I) in an interference-limited environment. This, of course, is due to the need for only a binary decision in most signal detection systems. As binary information is coded into higher order systems (to minimize bandwidth for transmission), the digital system more closely approaches typical analog systems in their tolerance to interference. For reasons discussed in more detail below, multiple-level system do not appear to be appropriate for operation in an interference-limited environment, hence, it appears desirable to limit the investigation to modulation techniques that employ no more than four states for phase and frequency modulation.

2.1 Choices of Modulation Techniques

We generally assign modulation techniques into simple categories such as amplitude modulation (am), frequency modulation (fm), and phase modulation or phase shift keying (pm or PSK). However, it is important to recognize that, for digital systems at least, the detection technique to be employed is of equal, if not greater, importance when considering system performance. Further, when detection techniques are considered, some systems tend to lose their identity with the simple definitions above. For example, most PSK systems are suppressed carrier amplitude modulation systems using coherent or differentially coherent detection, though PSK modulation techniques exist in which the generated signal has the characteristics of an fm signal, including its constant signal envelope. (For this reason, Hubbard[1] has further categorized PSK systems into am-PSK and fm-PSK systems.) Often the choice of technique is strongly influenced by equipment limitations such as peak power capability, am-to-pm conversion when amplifying variable level signals (ie, am-PSK), and spurious emission resulting from clipping of a variable-level signal.

When the transmitter characteristics are under consideration, we are most concerned with the envelope characteristics of the radiated signal. However, these notes are primarily concerned with reception. Therefore, a rather specific model has been adopted for the fm system. In particular, it has been assumed that the baseband detector process is separated from the digital decision process. That is, the fm system considered is a <u>baseband</u> system and

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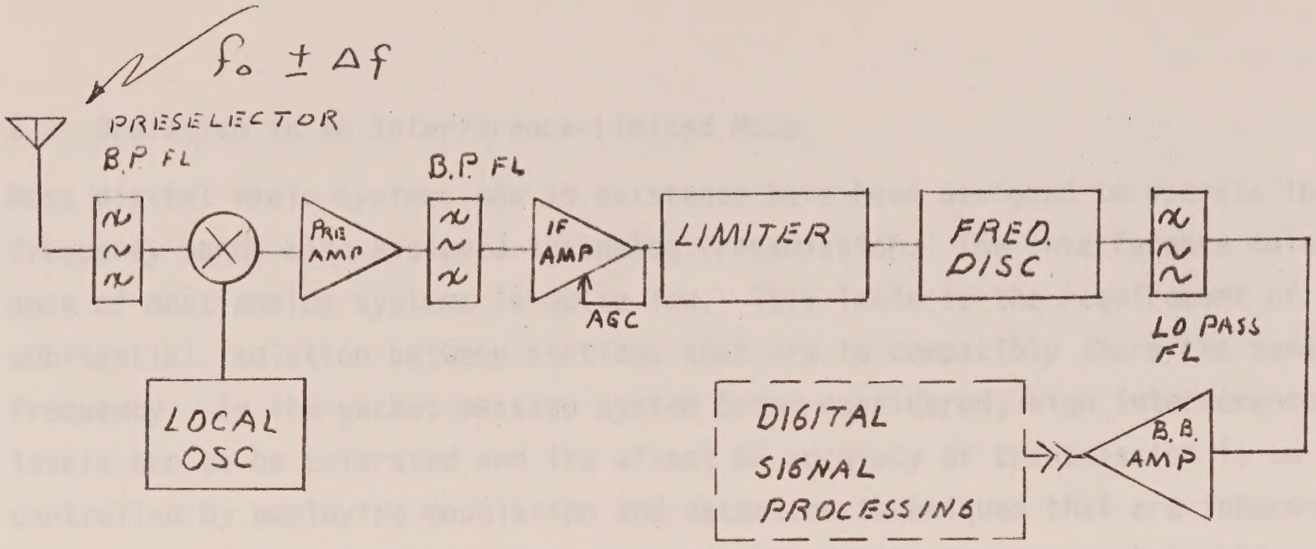
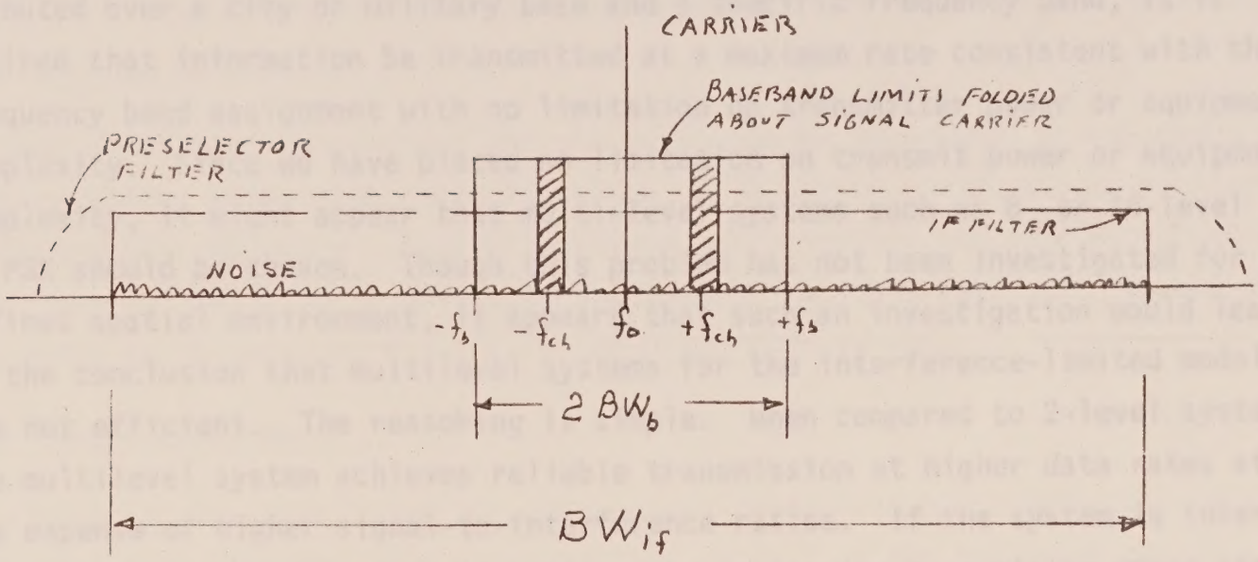
the purpose of the fm transmitter and receiver is to faithfully relay a particular digital signal which, then, may be further processed for optimum digital decision by integrate and dump, or some other technique. Such is not the case for the PSK system considered since digital decisions are made by processing directly the if signal in the receiver. Therefore, when the term S/N is used in considering PSK system, the signal considered is the total energy in the received signal, and the noise is the noise in the processing bandwidth in the receiver. However, for the fm system the term S/N is used to refer to the baseband noise of the demodulated signal in a defined baseband bandwidth, which may be much smaller than the total receiver bandwidth. For fm systems, the term carrier-to-noise is used to define the ratio of total signal power (ie, the same as unmodulated carrier power for fm) to total noise in the receiver bandwidth. It is hoped that the different use of the term S/N between fm and PSK systems will not lead to confusion. It is recognized that coherent detected fm systems do exist, but it is believed that such systems are appropriately defined as fm-PSK as suggested by Hubbard. Therefore, it is emphasized again that the fm system considered below is a baseband system that may be used with various types of digital signal processing equipment. For this reason, it is the S/N and not error rate that is given primary consideration in the evaluation of the characteristics for receivers.

2.2 Frequency Bands

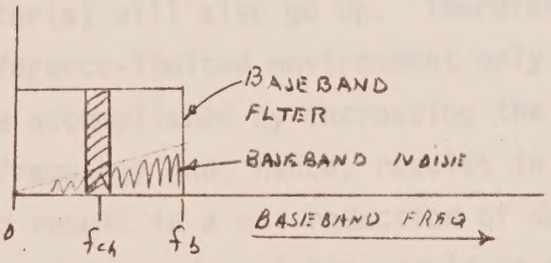
The choice of modulation technique does not appear to be influenced by the frequency band to be employed though the data rate to be employed may affect the choice of frequency band for bandwidth reasons. Operation in frequency bands from vhf through shf appears appropriate. It appears at this point that the choice of data rate will be influenced more by multipath propagation which, to some extent, may effect choice of frequency band. As packet transmission systems develop, special frequency band assignments are likely to be made in order to assure optimum operation in an interference-limited environment. Shared operation with other types of service, particularly analog, does not appear desirable. The limitation here, however, is primarily interference to other types of service rather than interference of such services to the packet transmission system. Industry filings in response to FCC Docket 19311 are particularly relevant to the interference issue, at least in the commercial microwave bands. Government band assignment probably presents less of a problem.



(a) FM Receiver Block Diagram.

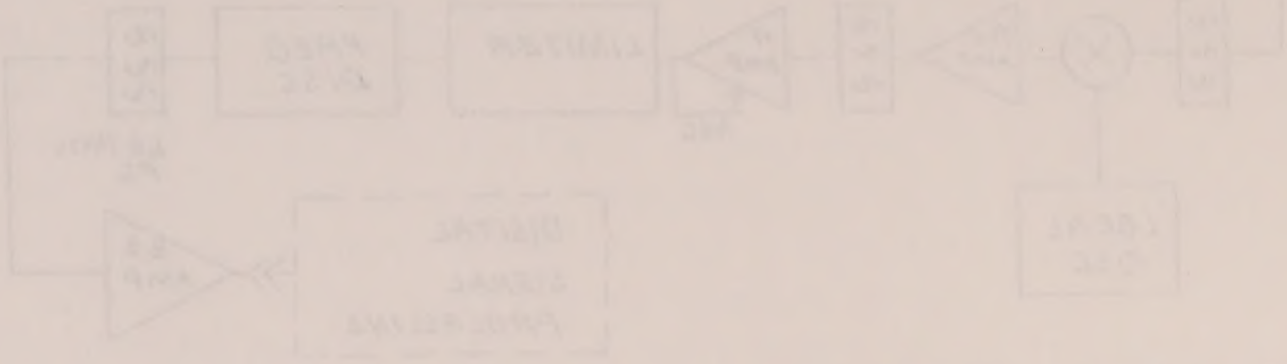


(b) RF, IF, and Baseband Filters



(c) Baseband Frequency Spectrum

Figure 3-1. Frequency Modulation Receiver Model Showing Relationship of Baseband, IF, and Preselector Filters.



2.3 Operation in an Interference-Limited Mode

Most digital radio systems now in existence have been designed to operate in frequency bands also assigned to analog transmission. The interference tolerance of most analog systems is quite low. This leads to the requirement of substantial isolation between stations that are to compatibly share the same frequency. In the packet message system being considered, high interference levels are to be tolerated and its affect on accuracy of transmission is to be controlled by employing modulation and detection techniques that are inherently tolerant to interference and by retransmission of the message packets when not correctly received. Therefore, the spatial environment is a very important consideration in the choice of modulation technique. Consider, in a general manner, the following situation. In a given spatial configuration with stations distributed over a city or military base and a specific frequency band, it is desired that information be transmitted at a maximum rate consistent with the frequency band assignment with no limitation on transmitter power or equipment complexity. Since we have placed no limitation on transmit power or equipment complexity, it might appear that multi-level systems such as 8- or 16-level fm or PSK should be chosen. Though this problem has not been investigated for a defined spatial environment, it appears that such an investigation would lead to the conclusion that multilevel systems for the interference-limited model are not efficient. The reasoning is simple. When compared to 2-level systems, the multilevel system achieves reliable transmission at higher data rates at the expense of higher signal-to-interference ratios. If the system is interference limited, this cannot be achieved by increasing transmitter power since the power of the interfering transmitter(s) will also go up. Therefore, we can employ multilevel systems in an interference-limited environment only by providing greater isolation. This must be accomplished by increasing the spacing between stations that share the same frequency and, hence, results in a reduced density of stations. The effect could result in a net reduction of data flow throughout a given network. Of course, increased isolation could be obtained in some situations by means of antenna directivity and this probably should be considered for some fixed stations. However, the increased station density to be obtained by this technique applies equally well to modulation systems of all levels, hence, should not influence the choice of modulation technique.

Another important consideration is the fact that the data rate increases only as $\log_2 N$, where N is the number of levels employed. Therefore, in going from

a 4-level to an 8-level system at a given symbol rate we only increase the bit rate by 3/2. However, with the 8-level system, filter characteristics become more critical because of intersymbol interference, hence, in many practical systems which may be limited by filtering, we may need to lower the symbol rate when going to an 8-level system, and may not even achieve the 3/2 bit rate advantage. As we consider even higher levels, the problem only gets worse; therefore, the theoretical 4/3 bit rate advantage in going from 8 to 16 levels is almost certain to be reduced to no advantage at all when practical hardware limitations are considered, even if the system is not to be operated in an interference-limited environment. So, emphasis in the planned investigations will be given to simple modulation and detection systems of no more than four states. Such an approach also has the result of simplifying the equipment, which is a major consideration where a large number of stations are involved.

2.4 Maximum Data Rates

Maximum data rates permissible are not likely to be determined by equipment considerations since data rates in excess of 10 Mbit/s are practical with simple and economical hardware. A more important consideration influencing choice of data rate is likely to be multipath propagation characteristics of the uhf and shf bands to be employed for the system under consideration.

2.5 INTERFERENCE MODEL

In attempting to establish error rates under conditions of interference it is important that the amplitude distribution, as well as the rms power of the interfering signal be defined. The basis of this statement is probably obvious. In most digital transmission systems we are interested in low error rates (ie, at most, only one error per 1000 bits transmitted for system using error correction and often error rates of 10^{-6} to 10^{-8} are required). If the interfering signal has a high peak-to-rms value, then the error rate is almost totally determined by the statistics of the peaks of the interfering signal. Most investigations of data system error statistics have been concerned with the weak signal case where the interference is thermal noise. For this case, a Gaussian distribution is assumed. When the interference is a relatively constant envelope signal (low peak factor), such as a single CW, FM, or PSK signal, the error rate characteristics can be quite different from those resulting from an equal level of Gaussian noise. Examples of this are given

in section 5. However, if the system is operated in an environment of numerous interferors, then the peak factor again approaches Gaussian noise. Therefore, for the system being considered where a large number of interferors are expected, it seems reasonable to take Gaussian noise as a "worst-case" condition of interference. This, then, allows maximum use of extensive prior analysis of fm and PSK system performance when limited by Gaussian noise. The effect of interference is then equivalent to lowering the signal-to-noise ratio.

3.0 CHARACTERISTICS OF FREQUENCY-MODULATED SYSTEMS

An fm receiver model is defined below and characteristics for several sets of specific receiver parameters are presented. Emphasis is given to the threshold characteristics and a simple technique of defining the S/N and threshold performance of fm receivers is included as appendix I to these notes.

3.1 FM RECEIVER MODEL

A block diagram of a typical fm receiver is shown in figure 3-1. In this receiver the input signal is translated to an intermediate frequency to facilitate bandpass filtering and signal amplification. A preselector filter is included prior to the down-converter to reduce image response and high-order mixing products from adjacent channels to an acceptable level. The mixer is followed by a low-noise amplifier which may contain some bandpass limiting. The if filtering is placed prior to the main gain of the if amplifier. The main if amplifier contains a wide-range agc so that constant level is presented to the limiter. The receiver must have sufficient gain prior to the limiter to maintain the input to the limiter constant, even with no input to the receiver (ie, full limiting on the receiver thermal noise), if the threshold performance shown in the examples below is to be achieved. The limiter is followed by a discriminator whose output is a linear function of the deviation of the instantaneous signal frequency from the center frequency. The discriminator is typically followed by a baseband amp. Other variations of the typical receiver may be used; for example, systems that operate at frequencies below 1 or 2 GHz may employ low-noise transistor preamplifiers prior to the down-converter. Microwave preamplifiers such as paramps and tda's probably have no value in the system being considered for economic reasons, but would only affect the carrier-to-noise, hence, is not of importance for the definition of threshold and S/N as a function of C/N. Assuming adequate gain, the receiver characteristics of interest are:

a. Receiver bandwidth (rf and if) prior to the limiter
b. Ratio of receiver rf and if bandwidth to received baseband width
c. Ideal limiter and discriminator characteristics.

In a practical system the limiter characteristics are particularly significant in determining threshold performance since most discriminators are amplitude sensitive; hence, require a limiter to remove the am component of the noise.

The various parameters for the receiver model shown in figure 3-1 are defined as follows:

f_o = carrier frequency (assumed to be centered in the receiver bandpass)

Δf = peak deviation of fm carrier

f_{lo} = local oscillator frequency

BW_{if} = bandwidth of the if amplifier (which, for this typical model, controls the receiver bandwidth)

BW_b = bandwidth of low-pass baseband filter

f_b = maximum top baseband frequency (ie, cutoff frequency of baseband LP filter)

C/N = unmodulated (or total mod signal) carrier-to-total-noise ratio referenced to the discriminator input. Where noise bandwidth is not specified, it should be assumed that total receiver if bandwidth is intended.

$*f_{ch}$ = baseband frequency of some defined channel or bandwidth segment within the baseband

$*BW_{ch}$ = bandwidth of some defined channel or bandwidth segment within the baseband.

$\left.\frac{S}{N}\right|_T$ = signal-to-noise threshold defined as C/N at which S/N departs 3 dB from linear S/ N versus C/N relationship. Note that this definition is an arbitrary though convenient one. (See appendix I for further details on this definition.)

$\left.\frac{S}{N}\right|_{ch}$ = signal-to-noise of a particular baseband segment (ie, channel)

$\left.\frac{S}{N}\right|_b$ = signal-to-total-baseband-noise.

*It is hoped that the introduction of the concept of a baseband channel here does not lead to confusion since typically, the data signal for systems of interest will occupy the total baseband. The purpose in introducing this concept is to show in the following examples that both the threshold and the baseband noise spectral density are a function of baseband frequency for fm systems. If one chooses, the channel bandwidth may be selected as 1 Hz and the reference deviation as 1 Hz peak, then the resulting curves would be plotted in units of signal-to-noise spectral density referenced to 1-Hz deviation.

3.2 SIGNAL-TO-NOISE AND THRESHOLD CHARACTERISTICS

Consider the fm receiver modeled in figure 3-1; it is primarily the if band-pass filter and the ratio of bandwidth of the if filter to baseband filter that is of interest when evaluating threshold performance. For any system, the if bandwidth will be somewhat greater than twice the bandwidth of the base-band filter and for wide deviation systems, the if bandwidth will be several times larger than the baseband filter bandwidth. The Carson's Rule bandwidth of the receiver is $BW_{if} = 2(f_b + \Delta f)$, where f_b = the maximum modulating frequency, and Δf is the peak deviation. This value may generally be taken as the minimum receiver bandwidth that will provide adequate performance, and since it is difficult to design if band-limiting filter with linear phase to the band edges, a well-designed receiver will often require somewhat wider bandwidth. The controlling factor is the linearity (intermodulation distortion) that may be tolerated by the system.

With the above model in mind, consider the signal-to-noise characteristics of selected segments of the baseband (ie, baseband channels). If we sample segments of the baseband and measure the noise power density as a carrier is inserted (ie, measure quieting), we will measure a characteristic as shown in figure 3-2. It is seen that the degree of quieting as a function of C/N is a function of the baseband frequency, and that the "break" from the flat baseband quieting moves progressively lower in frequency as the C/N increases. Harris[2] has used the point of break from flat quieting to define a technique of deriving the S/N characteristics of any fm receiver. The technique is described in appendix I.

The above characteristic is shown in a more familiar form in figure 3-3, where the S/N is plotted as a function of C/N for a particular baseband channel bandwidth (3 kHz), channel deviation (234 kHz peak), and if bandwidth (15 MHz). Of particular interest in such curves is the threshold characteristic.

The above has been included in these notes to emphasize that for fm systems, both the baseband noise spectral density and the location of the threshold break are a function of baseband frequency for a particular system, and for the general fm receiver the threshold point is related to f_{ch}/BW, where f_{ch} is the baseband frequency of a particular channel, and BW is the receiver bandwidth.

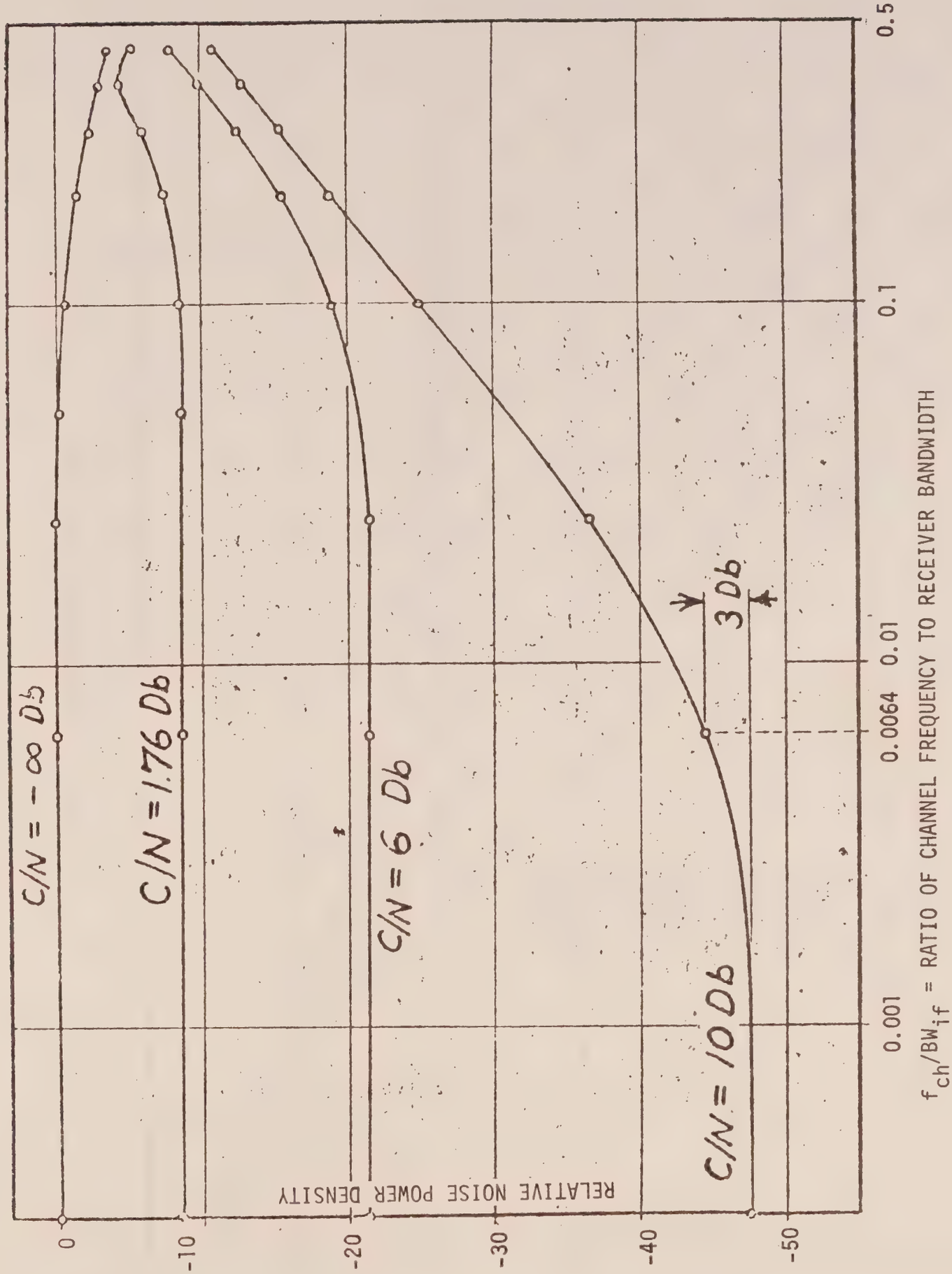


Figure 3-2. Plot of Noise Quieting Characteristics of a Typical FM Receiver.

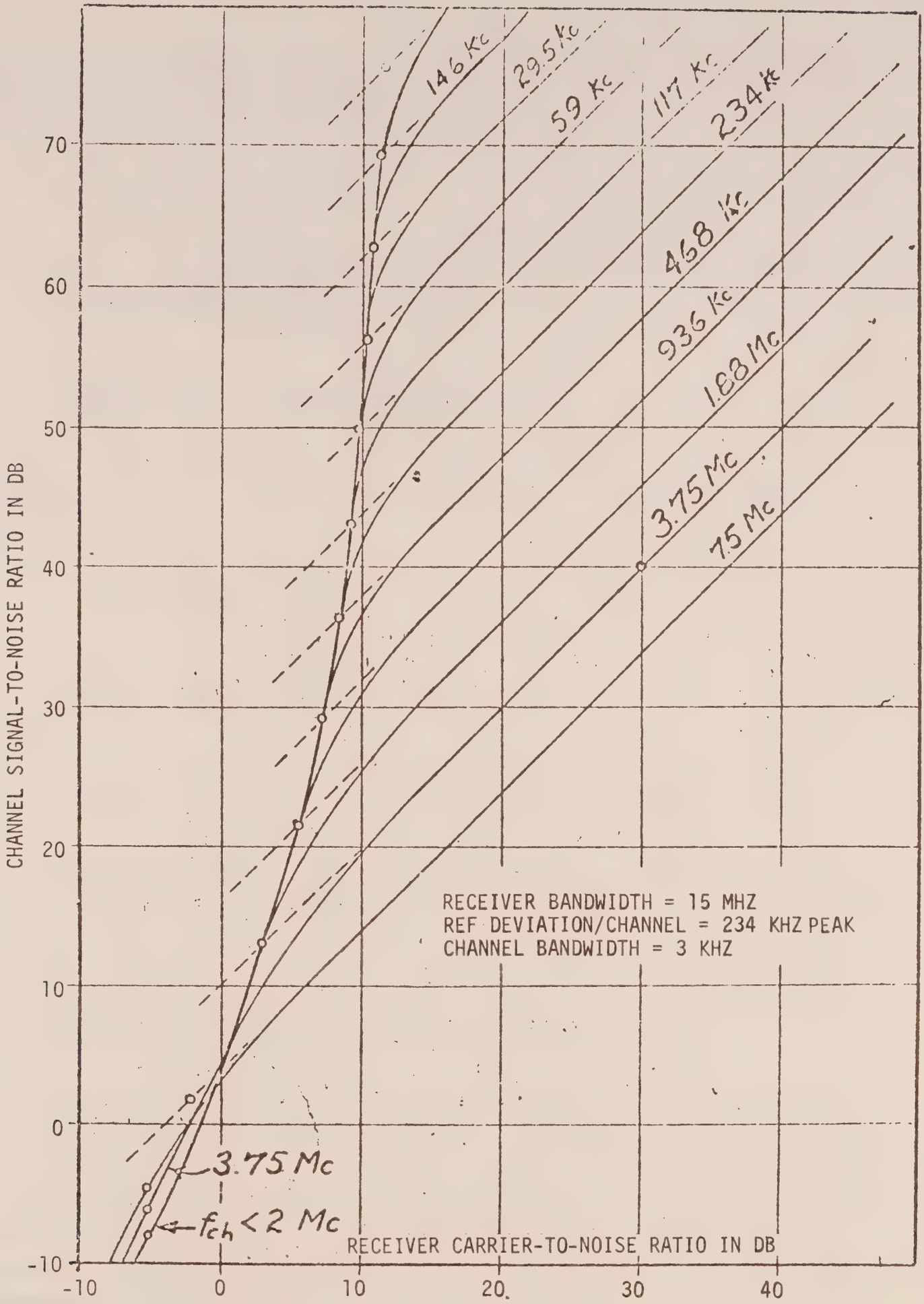


Figure 3-3. Baseband Channel Versus S/N for a Typical FM System Used for Relaying Multichannel Telephony Signals.

Once the C/N is above the threshold, the S/N of a particular baseband segment or channel is defined by the relation:

$$S/N_{ch} = \left(\frac{C}{N}\right)_{2BW_{ch}} \cdot \left(\frac{\Delta f}{f_{ch}}\right)^2$$

$$S/N_{ch} = \left(\frac{C}{N}\right)_{if} \left(\frac{BW_{if}}{2BW_{ch}}\right) \left(\frac{\Delta f}{f_{ch}}\right)^2$$

$$S/N_{ch} = \left(\frac{C}{N}\right)_{if} + 10 \log\left(\frac{BW_{if}}{BW_{ch}}\right) + 20 \log \frac{\Delta f}{f_{ch}} \quad .$$

If we are interested in the signal-to-total-baseband-noise ratio, this is defined by

$$S/N|_b = 3\left(\frac{C}{N}\right)_{2f_b} \cdot \left(\frac{\Delta f}{f_b}\right)^2$$

$$S/N|_b = 3\left(\frac{C}{N}\right)_{if} \cdot \left(\frac{BW_{if}}{2f_b}\right) \cdot \left(\frac{\Delta f}{f_b}\right)^2$$

$$S/N|_b = 4.77 + (C/N)_{if} + 10 \log \left(\frac{BW_{if}}{2f_b}\right) + 20 \log \left(\frac{\Delta f}{f_b}\right) \quad \text{dB} \quad .$$

The above relationships are valid for C/N sufficiently large that S/N is a linear function of C/N (ie, well above threshold). It will be noted that for this condition, the S/N of the channel is independent of the if bandwidth. Therefore, increasing the deviation ratio, $\frac{\Delta f}{f_{ch}}$, improves the S/N 6 dB each time the deviation is doubled. However, increasing the deviation ratio does move the threshold point as shown in the example in figure 3-3.

The significance of the above can be easily understood by referring to the bandwidth relationships shown in figure 3-1. The baseband noise is determined only by the noise in the if receiver bandwidth that occupies slots equal to the channel bandwidth at $\pm f_{ch}$ from the signal carrier. The reason this is the case is the noise level is dominated by the product of these two noise components against the much stronger carrier. The product of any other noise components will be much lower than the two that lie $\pm f_{ch}$ from the carrier and,

hence, may be neglected. This concept also leads to a simple technique for defining the interference at baseband for specific interference components that may occupy the desired channel. This subject is discussed further in section 3.3.

The relationships between S/N, C/N, and threshold for three receivers are shown in figure 3-4. The following parameters were chosen for these examples:

For all receivers:

Noise figure = 8.2 dB

Top baseband frequency = ~~1 MHz~~ 100 KHz

Channel frequency in baseband = 50% kHz (ie, 1 octave below top channel frequency)

Channel bandwidth = 1 kHz

For Receiver No. 1	For Receiver No. 2	For Receiver No. 3
Deviation = 500 kHz peak	Deviation = 1 MHz	Deviation = 5 MHz
Bandwidth = 1.2 MHz	Bandwidth = 2.2 MHz	Bandwidth = 10.2 MHz

*Bandwidth for each receiver is Carson's Rule bandwidth.

For these examples, a relatively wide deviation ratio of 5* was chosen for even the lowest deviation ratio channel, hence, the receiver bandwidth required is almost a linear function of deviation ratio. Since the noise distribution over the channel is not flat, but rises 6 dB/octave, a relatively small channel width of 1 kHz was chosen so that error in S/N due to nonflat baseband noise spectral density would be small.

It is seen from the example in figure 3-4 that increasing the deviation ratio by a factor of 10 results in a strong signal S/N improvement of 20 dB. The price paid for this improvement is an occupied bandwidth increased by a factor of almost 10, and a loss in threshold in the neighborhood of 10 dB. If the threshold is defined as the point where S/N departs 3 dB from the linear relationship, then the loss in threshold is about 12 dB. However, the definition is a rather arbitrary through convenient one. For the highest deviation ratio system shown in figure 3-4, the break occurs at a S/N that is almost 30 dB higher than that for the receiver for one-tenth this deviation. Hence, for many types of systems the channel will be usable well below threshold break for the higher deviation ratio channel. This is not necessarily true for fm systems transmitting digital data if low error rate is a requirement.

* Relative to top baseband frequency. Deviation ratio for lower channel plotted is 10.

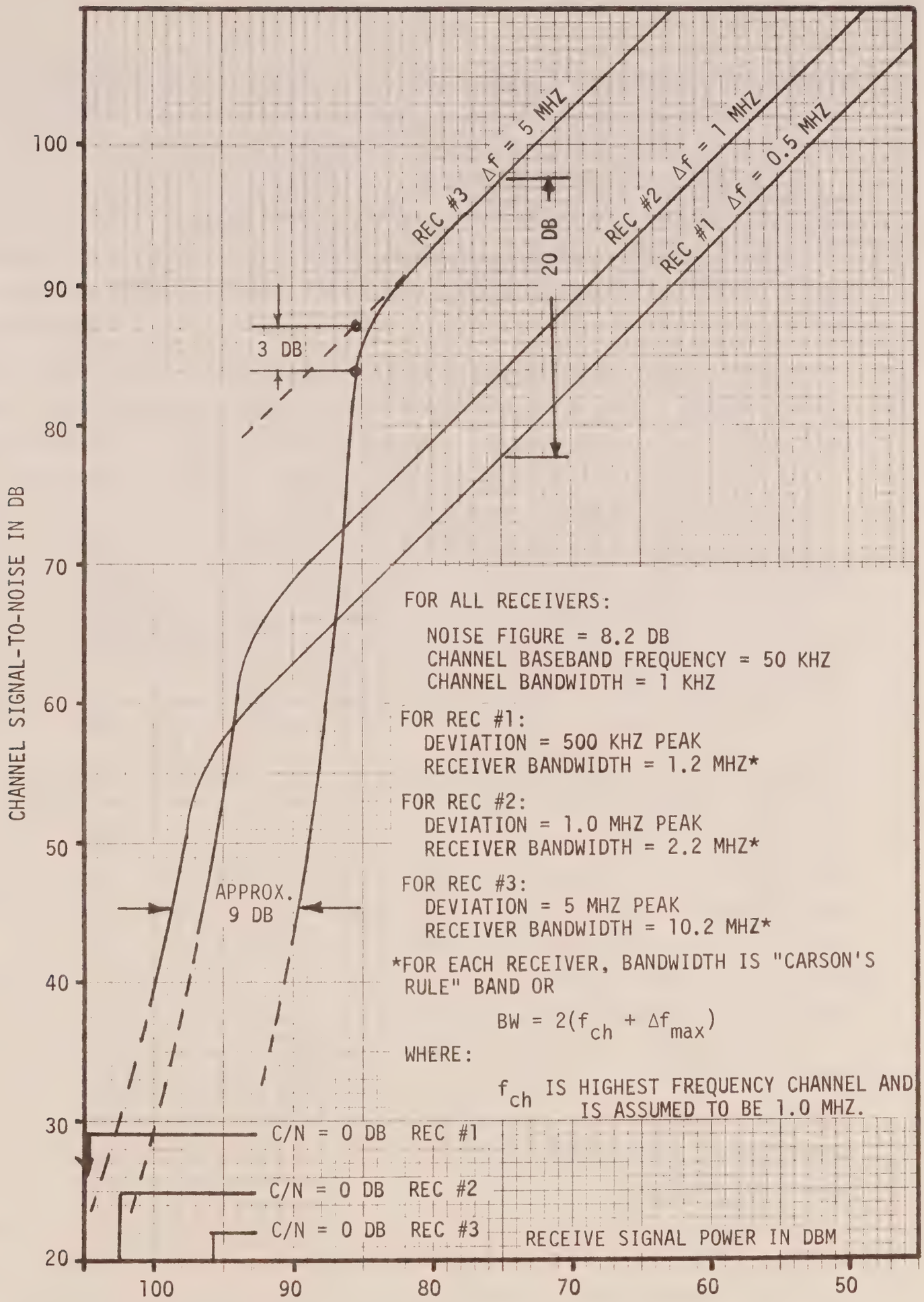


Figure 3-4. Signal-to-Noise and Threshold Characteristics of Three FM Receivers of Different Deviation and Bandwidth.

It has been shown by Cohn and Rice that the threshold is caused by noise peaks that exceed the level of the carrier at a phase such as to cause addition or subtraction of one complete rotation of the carrier vector. The "clicks" or "pops" that result in an fm system are the major contributors to the error rate of digital fm systems, hence, defining the system threshold in this manner is probably not a bad one for data transmission systems; further, since S/N drops rapidly below this point, the error from this approach could not be large in any event.

In the above discussion the receiver noise characteristic defined was for a channel occupying only a small part of the baseband spectrum. This was done for convenience in deriving the S/N curves. Since the data signal will occupy the total baseband spectrum, such an approach might be questioned. However, it will be noted from figures 3-2 and 3-3 that the baseband noise spectral density increases 6 dB/octave once C/N is above threshold. Hence, the total baseband noise power will be dominated by the last octave of the receiver baseband and, therefore, the shape of the receiver S/N characteristic would not change substantially from that shown in figure 3-4 if total baseband S/N were plotted. The primary effect is somewhat more rounding of the threshold break since the break point is both a function of baseband frequency and C/N.

It is hoped that the above aids in interpreting the capture ratio of fm systems. Referring again to figure 3-1, it will be noted that once S/I is above the defined threshold level, it is the interference that lies within $\pm f_b$ that determines the baseband noise resulting from interference. If we compare systems in which the desired and undesired signal have identical characteristics, it is seen that in the presence of interference, we gain S/N from two sources as deviation is increased. First, we obtain the S/N improvement resulting from the increased deviation ratio, and second, as we increase the deviation ratio of the interfering signal, the interfering energy that lies within band $f_o \pm f_b$ decreases. Therefore, if we increase the deviation ratio (and hence, bandwidth) of the interfering and desired signals simultaneously, the baseband noise due to interference will drop faster than the noise due to receiver thermal noise; hence, the "capture effect" can be quite pronounced in some analog fm systems. However, we must approach this with some caution in practical applications since the signal spectral density of the interferor over the band $f_o \pm f_b$ is a major factor in determining interference. Further, in many systems it is the carrier level of the interfering signal that may determine the interference

since the carrier stability of the desired and interfering signals may be such that the carrier "beat" may fall in the most important part of the baseband. Further, although the interfering signal may have a high maximum deviation ratio, the statistics of the transmitted signal may still result in a large percentage of the signal in the carrier or in strong sidebands near the carrier when measured over a long period of time. Examples of spectra from typical fm signals are shown in figures 3-5, 3-6, and 3-7.

It is emphasized that the above, relative to interference, is somewhat an oversimplification, though accurate for low deviation ratio fm signals (ie, where carrier level does not change appreciably, such as the spectrum shown in figure 3-5).

It is recognized that the above has not completely resolved the question of capture effect as a function of fm deviation ratio, but it can at least be qualitatively stated that "if the interference can be modeled by Gaussian noise, the threshold break point, when related to receiver C/N, is in the region of 8 to 12 dB and increases with deviation ratio. The rate of increase is slow, relative to bandwidth and deviation ratio increases, but for practical systems it is necessary to add the loss in C/N which results from the required increase in receiver bandwidth."

It appears that whether the above answers the question relative to capture ratio of digital fm systems lies in the answer to the following two questions:

a. Is the interference adequately modeled by Gaussian noise?
b. Is the "click" effect near the threshold break the controlling effect in determining digital system error rate?

It appears from analyses by Lucky that the answer for question b. is yes, at least for many systems that require very low error rate. Further, the click effect becomes controlling as the S/N begins to depart from the straight line relationship; hence, the real system threshold defined by a chosen minimum error rate may occur at a C/N above the 3-dB break threshold defined above. In such systems, increasing deviation ratio and, hence, bandwidth, only results in performance degradation.

3.3 SINGLE FREQUENCY INTERFERENCE

Where the desired signal contains a strong carrier component, the interference caused to fm systems by a single sinewave signal offset from the desired carrier can be computed by a simple technique. Though this is not particularly

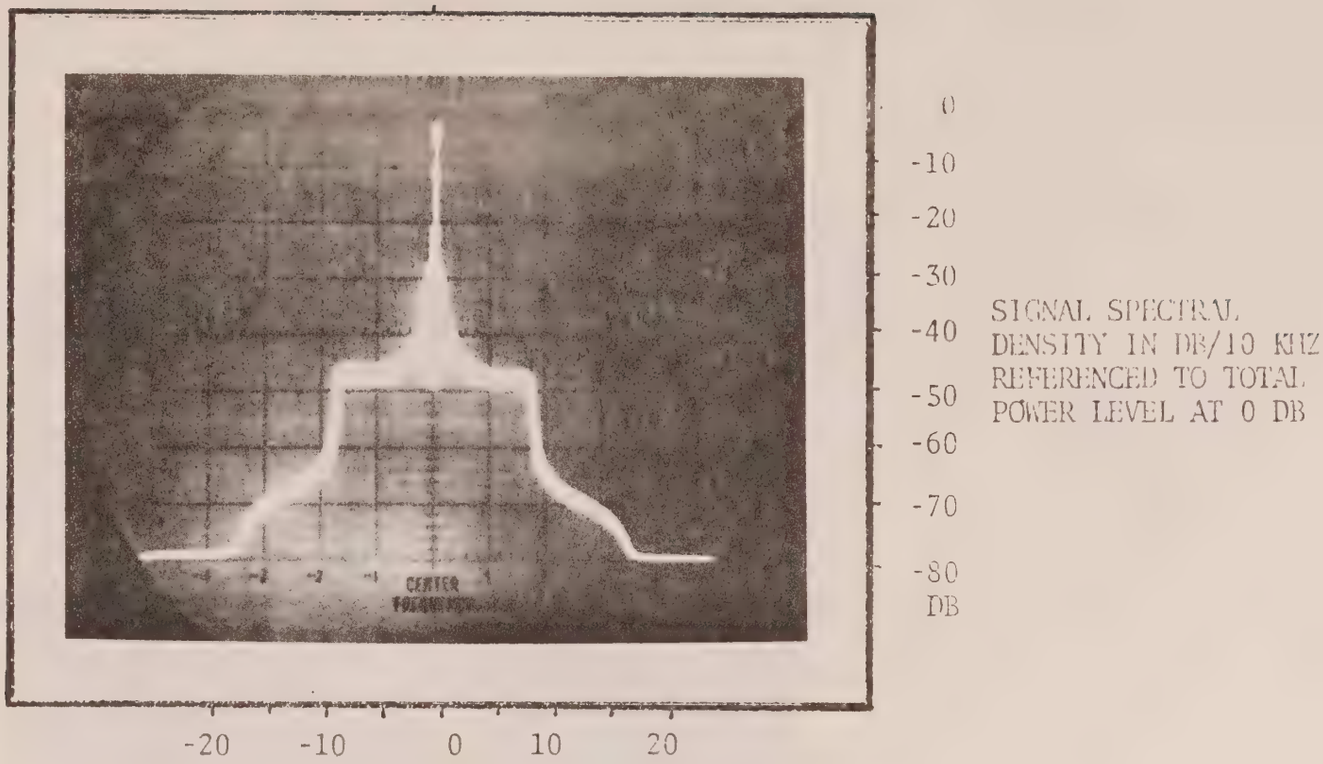


Figure 3-5. Signal Spectral of an FM Signal Modulated With Random Noise Distributed over Baseband of 300 kHz to 8.2 MHz (equivalent to 1800 SSB voice channels). Baseband signal has been emphasized 6 dB/octave over last octave. Note strong carrier component.

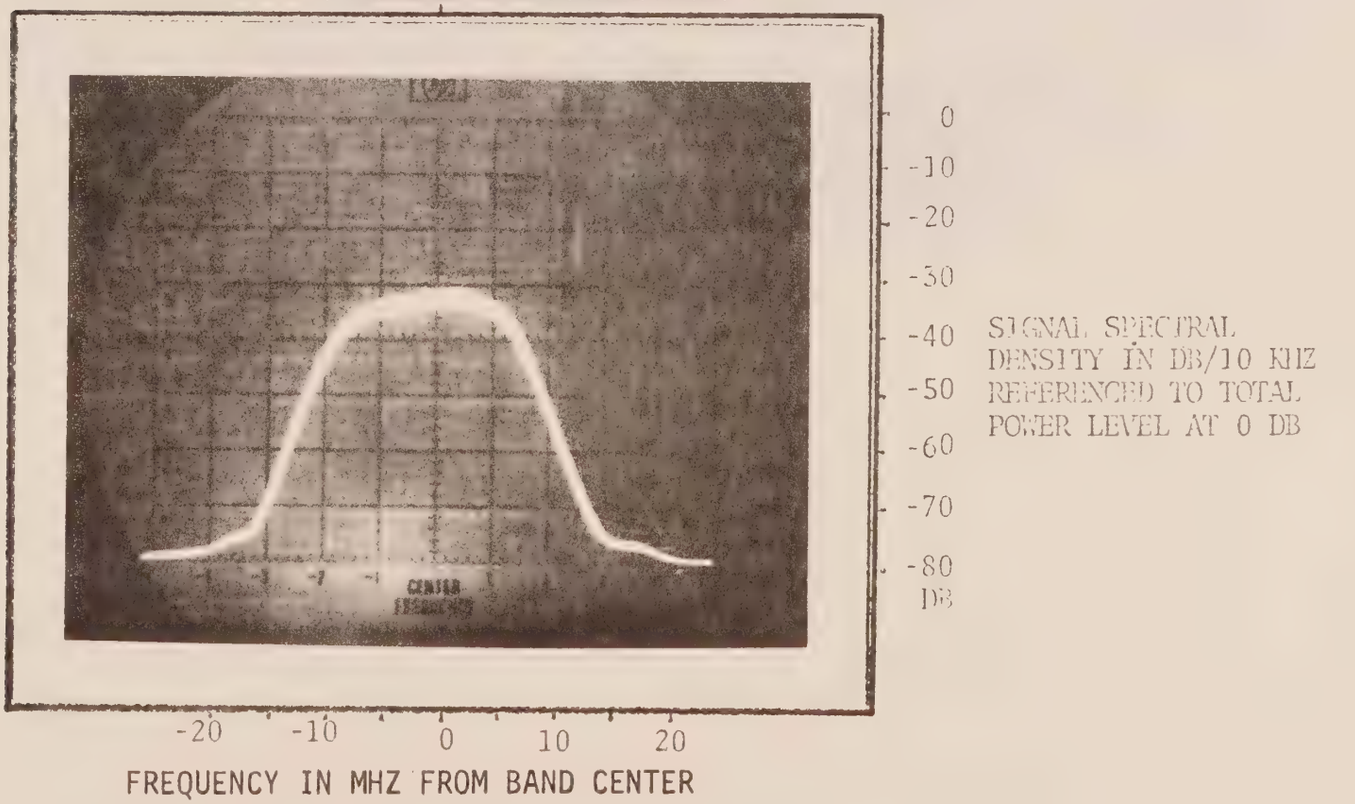


Figure 3-6. Spectral Density of a Filtered FM Signal With Random Data Modulation. Modulation Index = 0.5; Data Rate = 21.5 Mbit/s. Note absence of carrier component.

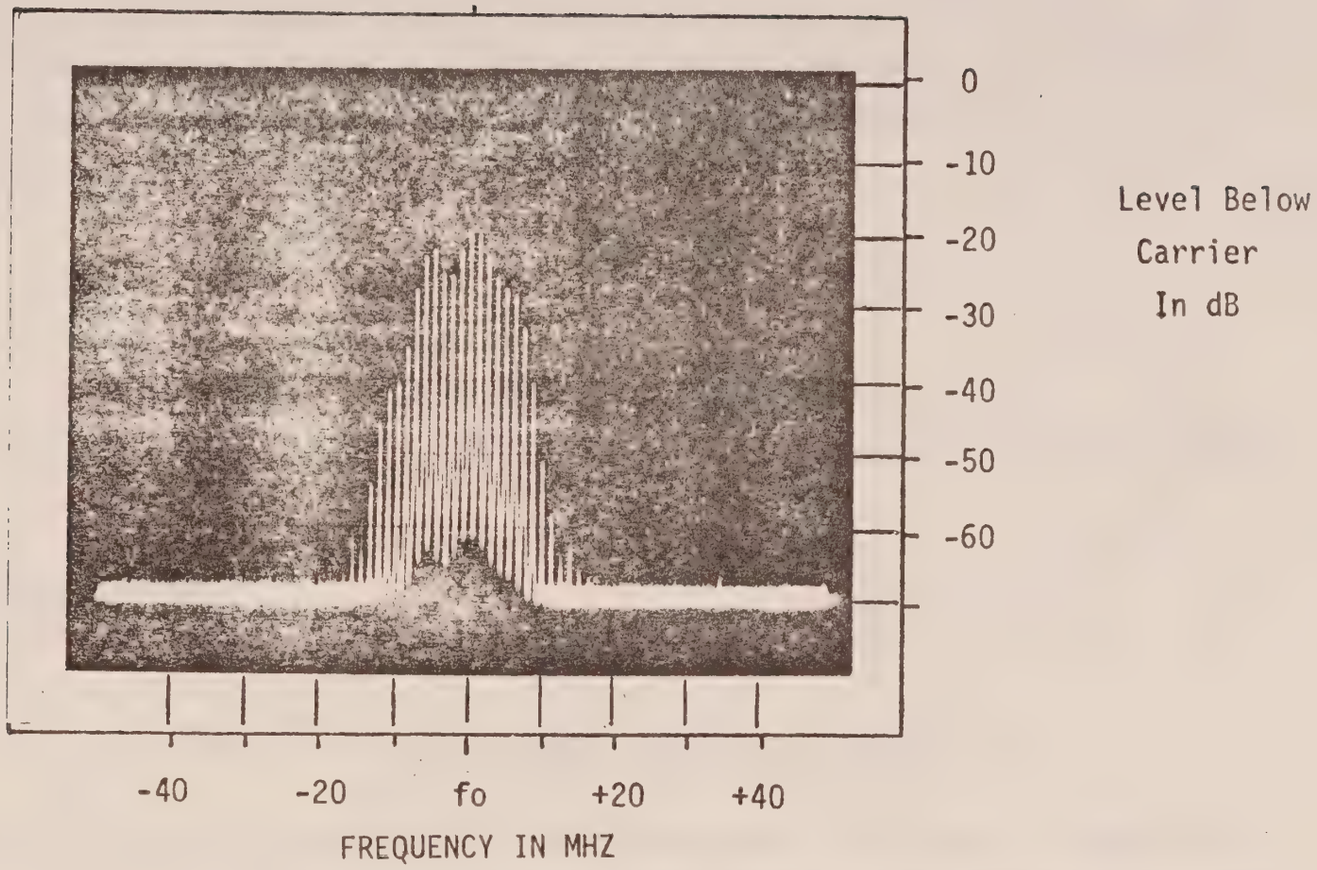


Figure 3-7. Filtered Spectrum of FM System Modulated With a 16-Bit Repeating Data Pattern. Modulation Index = 0.5; Data Rate = 21.5 Mbit/s. Note discrete sideband components that were absent for random data modulation shown in figure 3-6.

applicable to digitally modulated fm systems which may have no well-defined carrier component, this discussion is included here since it aids the understanding of the fm system and, in particular, the action of the limiter. Consider the spectrum and vector relationships of desired carrier and interference for situations where interference is well below the desired signal level as shown in figure 3-8 below.

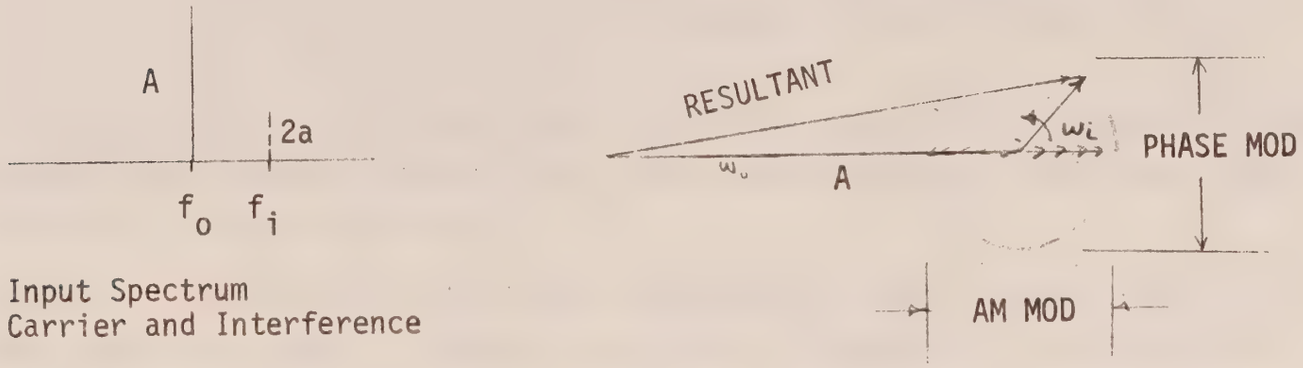
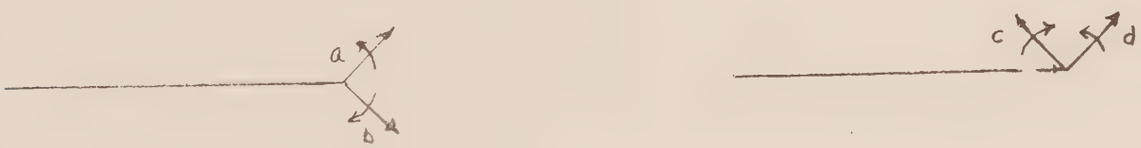


Figure 3-8. Vector Representation of Signal and Interference Prior to Limiting.

For this example, the amplitude of the interference is 2a and the amplitude of the carrier is A. It is seen that the system undergoes both amplitude and phase modulation due to the interference. We can assume that the above signal is a superposition of two vector systems below, one of which is pure phase modulation and one is amplitude modulation, which, added together, produce the original system (ie, a + d = 2a and b + c = 0)



(a) Amplitude Modulation (b) Phase Modulation

Where:

a = b = c = d, noting that amplitude of original interfering signal was 2a.

Figure 3-9. Division of Signal Plus Interference Prior to Limiting into AM and PM Components.

The purpose of the limiter is to remove the amplitude modulation, leaving only the vector system shown in figure 3-9b. It is seen that for interfering signals well below the level of the desired signal carrier the limiter operates on a nonsymmetrical sideband system in a manner to produce two symmetrical sidebands of amplitude 0.5, or each 6 dB below the original interferor. Further, if the interference is small, the peak phase deviation in radians is simply $\frac{2a}{A}$, since

$$\emptyset = \sin^{-1} \frac{2a}{A} \text{ and } \sin \emptyset = \emptyset \text{ (in radians) for small values of } \emptyset.$$

As the interference level approaches the carrier level, more sidebands are produced by the limiter in order to meet the constant amplitude requirement characteristic of phase modulation.

Examples of signal spectra and signal envelopes for both low and high interference levels are shown in figures 3-10, 3-11, 3-12, and 3-13. In viewing the output spectra it should be noted that the fm system used to obtain these spectra also included a deviation divider which divided the signal deviation by 4, following the limiter; hence, the sideband levels shown for the signal after limiting is 12 dB below that which would have been obtained if only a limiter were employed.

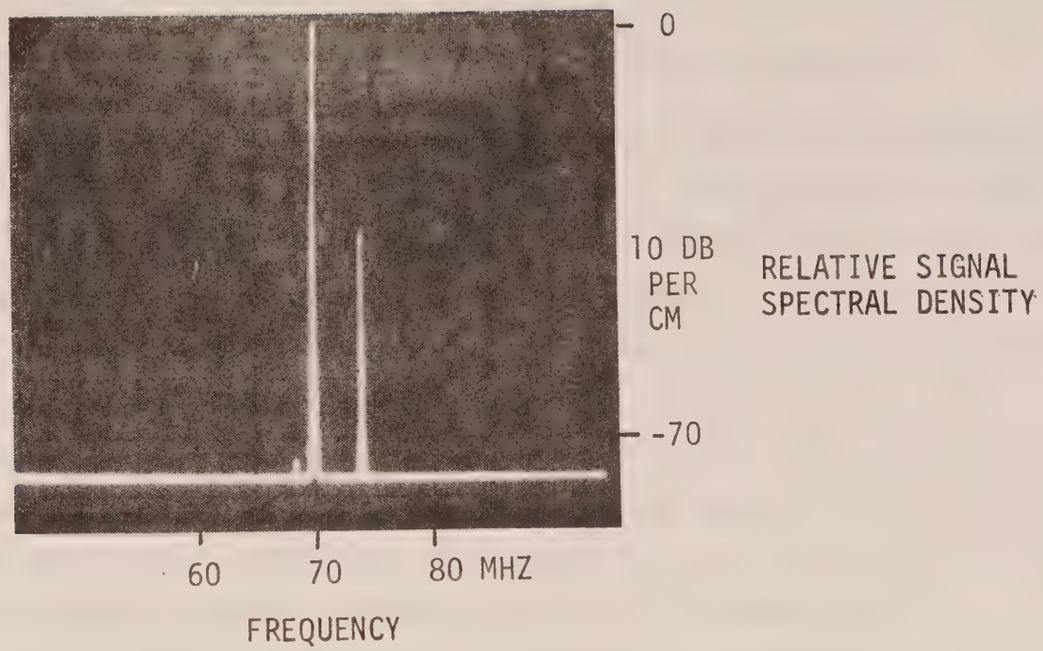


Figure 3-10. Spectrum of Desired Carrier at 70 MHz, and Interfering Signal Carrier at 74 MHz. Level of interfering signal is approximately 35 dB below desired signal carrier. This represents weak interference case.

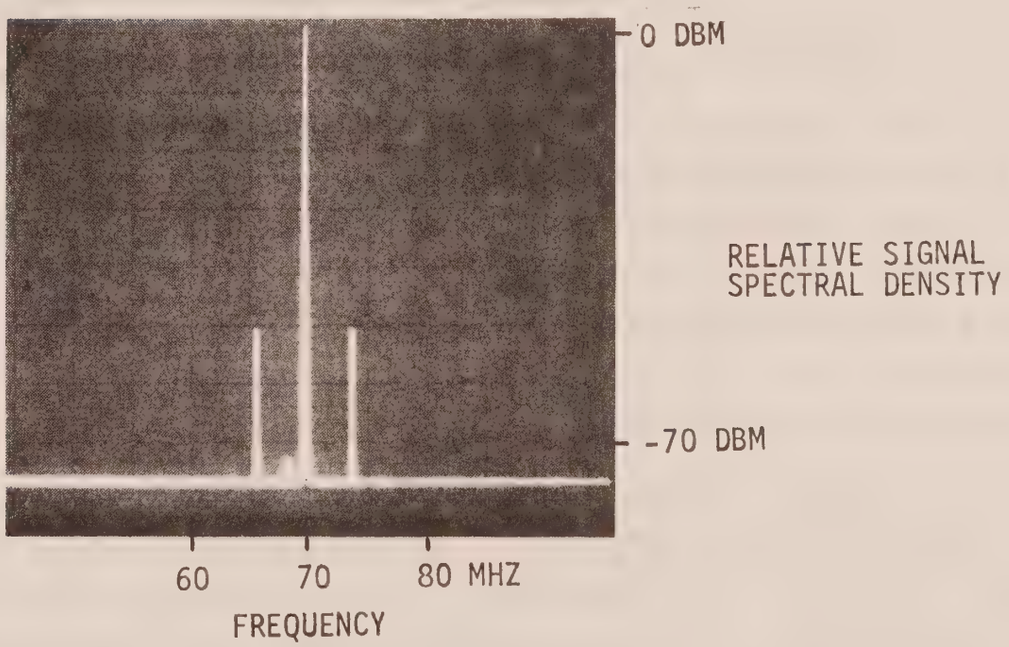
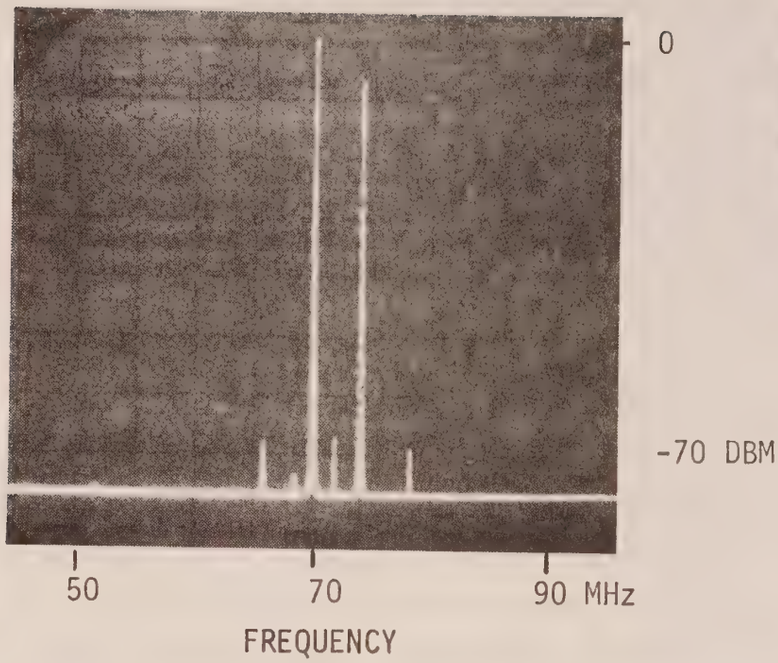
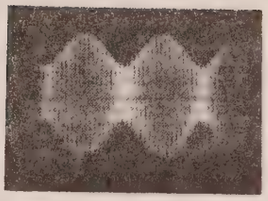


Figure 3-11. Spectrum at Output of Limiter Resulting From Interference Shown in Figure 3-10. Sidebands are symmetrical, indicating am component has been removed, leaving only phase modulation. Note that test configuration included a divide-by-4 deviation divider; hence, sideband level is 12 dB below that which would have been obtained from limiter alone.

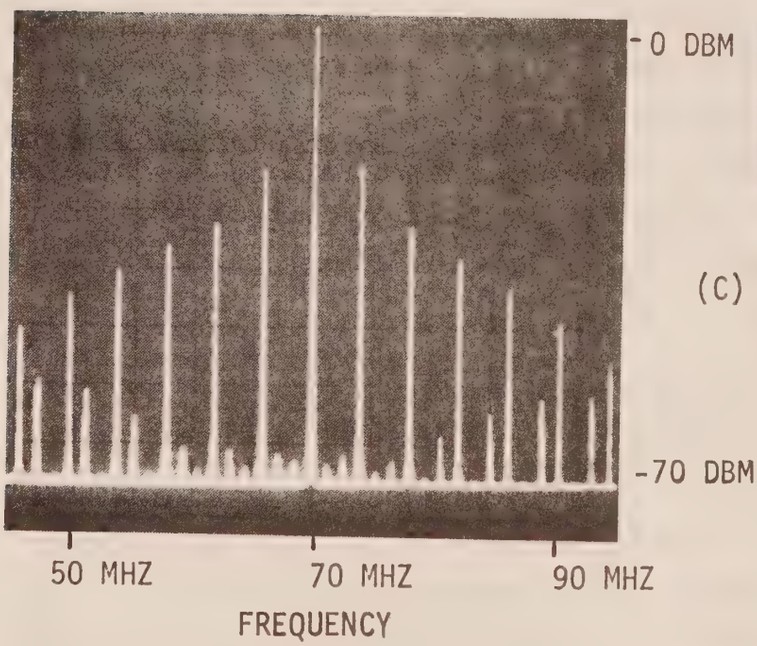


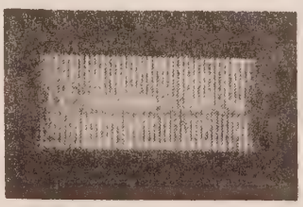


SIGNAL ENVELOPE

INPUT TWO TONE TEST SIGNAL. SPECTRUM AND SIGNAL ENVELOPE. 74 MHZ SIGNAL IS ABOUT 8 DB BELOW 70 MHZ SIGNAL.

Figure 3-12. Spectrum of Desired Carrier at 70 MHz and Interference at 74 MHz, at a level about 8 dB below desired carrier. (Signals at level of about -68 to -70 dB are spurious signals generated in the spectrum analyzer.)





OUTPUT SIGNAL ENVELOPE

(C) SPECTRUM AND ENVELOPE AT OUTPUT OF DEVIATION DIVIDER SHOWING ALMOST PERFECT SYMMETRY DUE TO LIMITING.

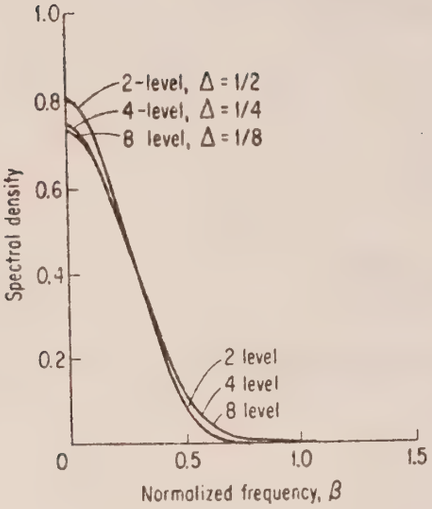
Figure 3-13. Spectrum at Output of Limiter for Input Signal Shown in Figure 3-12. Note symmetrical high-order sidebands produced as interference approach desired signal carrier level. (Test configuration included a divide-by-4 deviation divider following limiter, and this has reduced sideband level from that seen at output of limiter.)

3.4 Spectrum of FSK Data Signals as a Function of Deviation

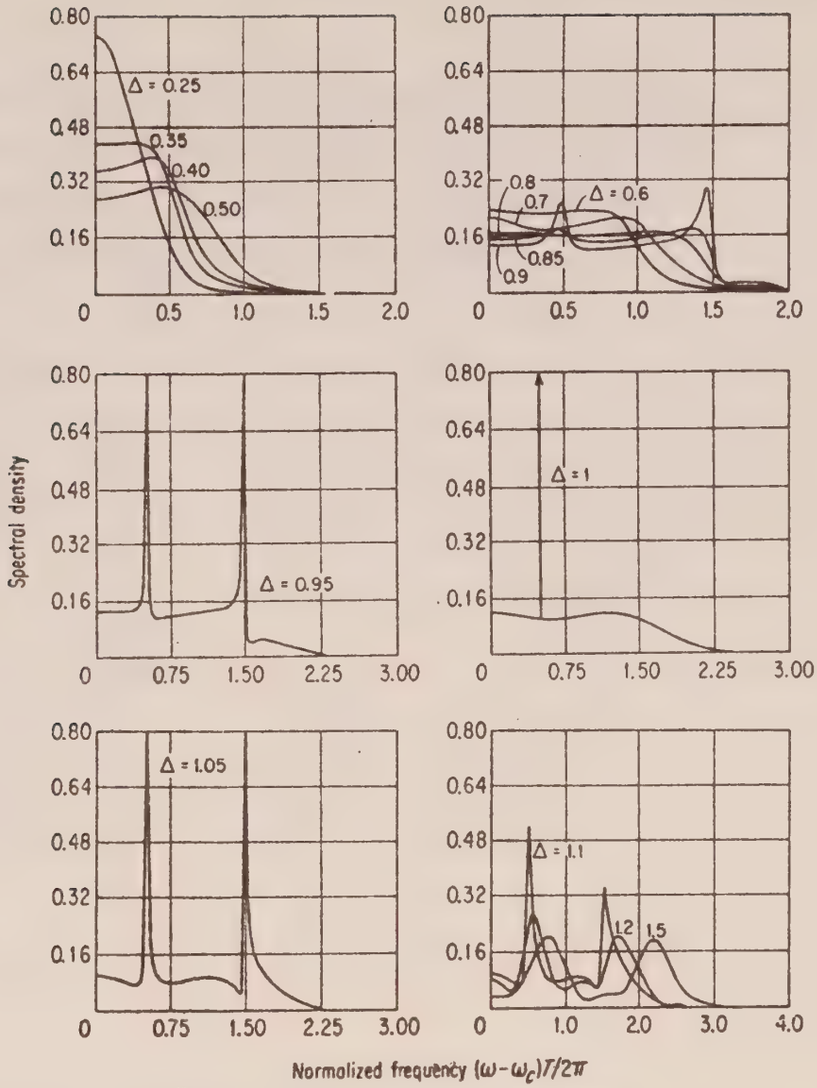
Lucky[4] and Bennett[5] have analyzed the spectral density characteristics of PSK data signals. When considering interference, the spectral density of the interfering signal is often quite important in its effect on the alowable level of interference for a given level of performance. Since the spectral density of PSK signals is quite dramatically affected by the modulation index, curves from Lucky are reproduced as figure 3-14. It will be noted that for binary fm (figure 3-14a) the spectral density of a signal with a modulation index of 0.5 and random data is a smooth, rounded distribution with no spectral peaks. As the index increases to 1.0, the spectral density becomes peaked at frequencies spaced from the unmodulated carrier frequency by ± 0.5 of the data rate. As the deviation is increased beyond an index of 1.0, the spectrum becomes increasingly complex and if high-deviation ratio fm (FSK) systems are considered, further analysis of the interference characteristics of such signals is warranted.

3.5 The "Click" Theory of FM Threshold

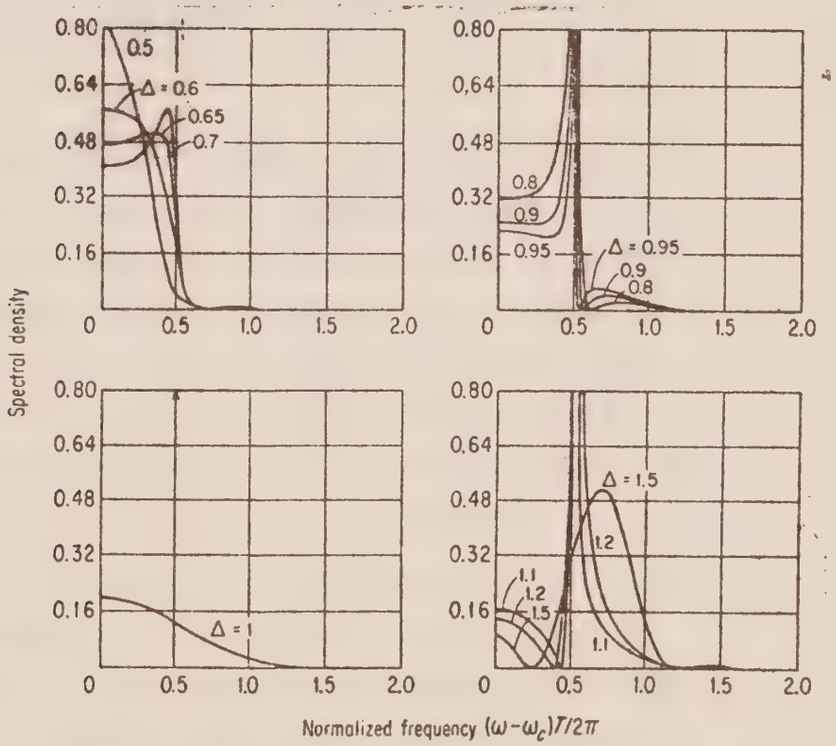
Rice[6] and Cohn[7] have introduced the theory of "clicks" to describe the threshold characteristic of fm systems. This theory has also been used to explain anomalies between the computed and measured error rate of practical fm systems. This subject has not been investigated in detail by the writer, and is introduced here only to emphasize that further investigation is justified since the question of potentially improved performance with increased deviation is closely tied to this subject. The term "clicks" has been applied to the acoustic effect observed when a wide deviation fm signal is faded into the threshold region. As the signal is faded, a series of random clicks or pops is heard, whose frequency of occurence rapidly increases as the signal strength is further lowered. Such phenomenon is also quite visually noticeable as impulsive noise in wide deviation fm tv transmission as used for satellite relay. Since these impulses are large in magnitude, they have the effect of increasing the peak-to-rms value of the receiver noise near threshold, and in a digital fm system will cause a greater number of errors than would have been computed from the baseband S/N alone, assuming Gaussian noise distribution. To better understand this phenomenon, consider the trajectory of the vector of the signal plus noise. The noise is continually changing in both amplitude and phase. Consider the trajectory of two noise bursts of essentially the same peak amplitude as illustrated by the vector diagrams in



C. Digital FM spectra for $\Delta = 1/L$.



b. Four-level digital FM spectra.



a. Binary digital FM spectra.

Figure 3-14. Signal Spectral Density as a Function of Deviation Ratio for Systems Modulated With Random Digital Data. (Data from Lucky.)[4]

figures 3-15a and 3-15b. Note that for 3-15a the peak phase error is somewhat less than $\pi/2$, while in figure 3-15b the noise burst causes one complete rotation of the desired signal vector. Therefore, a click occurs when the noise burst adds or subtracts one complete rotation of the signal vector.

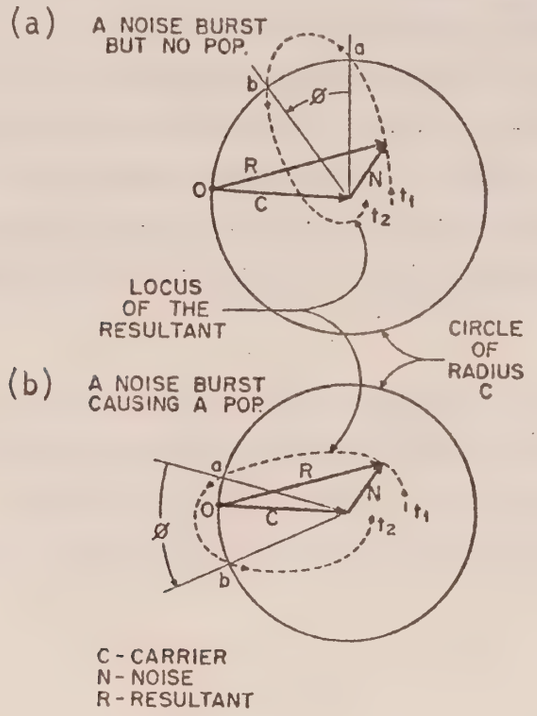


Figure 3-15. Vector Diagrams Showing Trajectory of Noise Bursts that Cause "Clicks" or "Pops." From Cohn.[7]

Recognize, of course, that the limiter maintains the total signal vector at constant amplitude. Since frequency is the rate of change of phase, it is obvious that noise trajectories such as shown in 3-15b result in large frequency excursions. Further, Cohn shows that disturbances such as shown in figure 3-15b have a nonzero average, while that of figure 3-15a has a zero average. Hence, noise burst such as illustrated by figure 3-15b result in a substantially low frequency component which explains the sound of the disturbance observed in the operation of audio channels near threshold.

Relations have been derived for the frequency of click occurrence as a function of C/N and other fm system parameters. It should be obvious that in narrow-band systems the rate at which a noise impulse can vary in amplitude and phase is limited by the receiver bandpass filter to a period comparable to the data symbol period. Therefore, in a narrow-band fm (FSK) system, a click is almost certain to cause a data error. However, as we increase

deviation and bandwidth, such is not the case and the impulse may be quite a bit shorter in duration than the data symbol and, hence, may not cause an error. Therefore, determining error rate is more complex than simply determining click rate. Further, the click rate is related to the position of the desired signal in the receiver bandpass and, hence, click rate is not necessarily the same for a modulated signal as for an unmodulated signal, and the frequency of occurrence of negative clicks is not necessarily the same as positive clicks for a modulated signal. There is also some indication that for wideband systems the clicks are not statistically independent and this increases the probability of a given click causing a data error in a wide deviation system.

Although it is relatively clear from the data reviewed that wide deviation fm systems do not offer capture advantage sought for the ARPA system, further investigation of the click phenomenon is believed to be justified so that error rates can be numerically defined in the region of fm threshold.

4.0 PERFORMANCE OF IDEAL SYSTEMS THAT EXPAND BANDWIDTH

In expanding upon theories developed by Shannon, Prabhu[3] has shown that a theoretical lower bound exists for S/N as a function of bandwidth expression factor, n, where n is defined as:

$$n = \frac{W}{W_o} = 2TW$$

where:

W = channel bandwidth

W_o = information bandwidth

T = Nyquist interval = $\frac{1}{2W_o}$.

For such a system, the data rate R is defined by

$$R = 2W_o \log_2 m \quad \text{bits/sec}$$

where:

m = number of characters that can exist during any symbol period (ie, number of levels or phases).

The lower bound of S/N versus n for arbitrarily low error rates from this reference is shown in figure 4-1. The condition of arbitrarily low error rates also assumes arbitrarily long processing times. Therefore, this curve should be interpreted as the theoretical performance that may be achieved with long processing times.

In many systems it is desired that the processing time be short. In particular, in the ARPA packet transmission system the processing time should not exceed the period required for the transmission of one data packet and it appears that for reasons of equipment simplicity it may be desired to limit processing time per symbol to no more than one symbol period. This case has been investigated by Prabhu and the limiting performance of the ideal system is compared to an fm system operated with a bandwidth expansion factor of 3. However, Prabhu avoids the threshold question by showing performance data for S/N well above 0 dB S/N. He specifically states that the equations from which the error rate performance is derived for the fm system is applicable only when S/N is much greater than unity.

4.1 Performance of Optimum System

The limiting error rate performance for systems which process the data over

only one signaling interval is shown for a 2-level signal for various levels of bandwidth expression in figure 4-2. Prabhu also shows limiting (ie, ideal) performance for systems with levels from 2 to 32. The required C/N for 10^{-6} error rate is plotted for systems with 2 to 32 levels and expanded factors of 3 and 17 in figure 4-3. It will be noted that even for the ideal system, bandwidth expansion techniques are most effective in improving performance for the multilevel system. For example, at an error rate of 10^{-6}, a 32-level system requires about 10.5 dB more S/N than a 2-level system for bandwidth expansion factor of 3; while for a system with bandwidth expansion of 17, the S/N necessary for a 32-level system is about 3.5 dB above that needed for a 2-level system. This agrees with one's intuitive evaluation of the situation as, for example, when considering fm since it would be expected that an increase in S/N obtainable by using high deviation should be most effective in improving error rate performance for multilevel systems which require high S/N to allow identification of the transmitted level in a multilevel system. However, as seen from data in figure 4-4, below the fm, (FSK) system performance is considerably poorer than the ideal system and the difference is greater for 8-level systems than for 4-level systems.

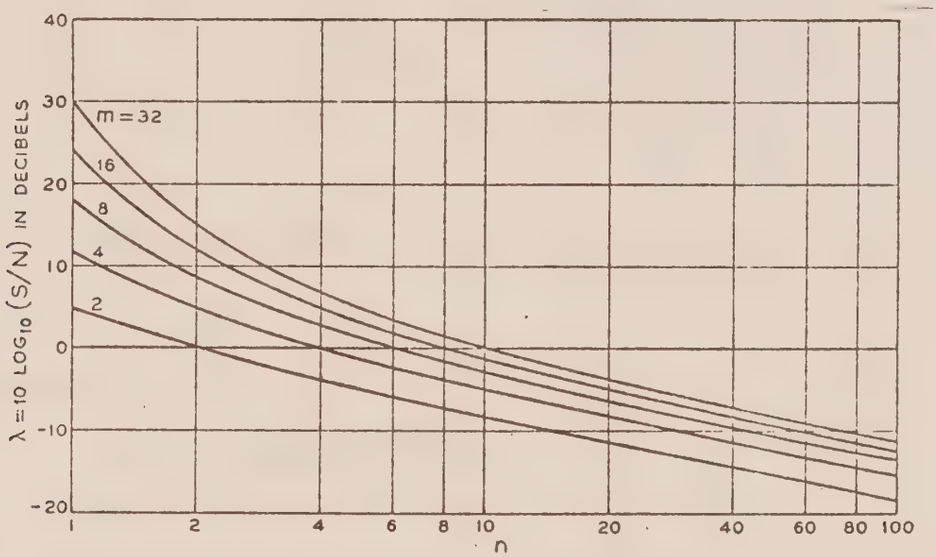


Figure 4-1. Lower Bound to the Signal-to-Noise Ratio for Different Bandwidth Expansions and Ideal Signaling.

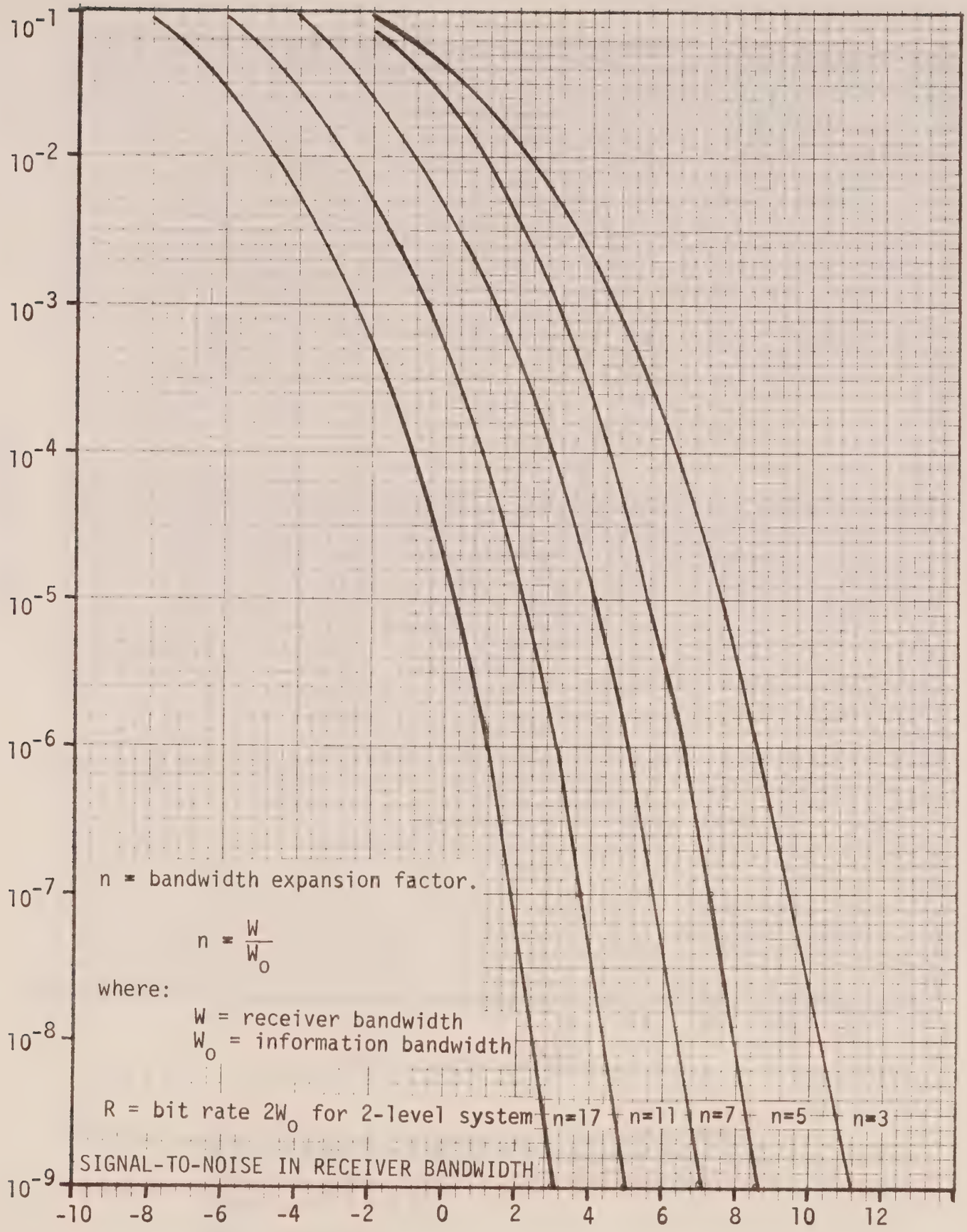


Figure 4-2. Lower Bound of Error Rate of Ideal System for Various Bandwidth Expansion Factors, n. Curves are for 2-Level System (m = 2).

Data from Prabhu.[3]

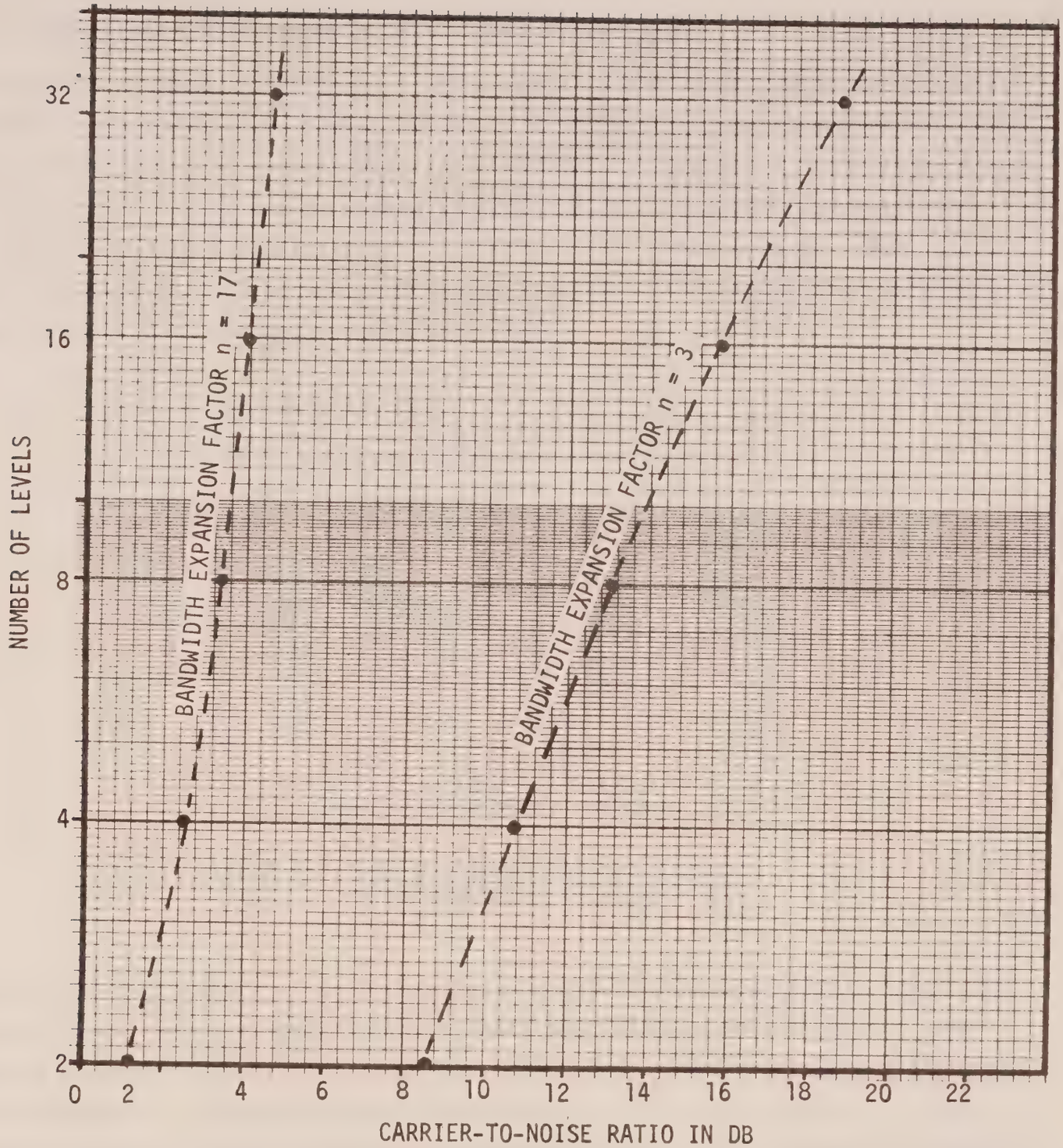


Figure 4-3. Required D/N for Ideal System for Data Error Rate of 10^{-6} with Maximum Processing Time Equal to Symbol Period. Data for expansion factors of 3 and 17.

Data from Prabhu.[3]

4.3 Comparison of FM to System With Ideal Performance

Prabhu has related 2- and 4-level FSK systems to the ideal system for bandwidth expansion of three, levels of 4 and 8, and a processing time that does not exceed one symbol period, T. These data are presented in figure 4-4. It will be noted for a 4-level system the fm system performance is 8 to 9 dB below the ideal system and for an 8-level system the differential is about 13 dB. Data are not plotted below 10 dB S/N and error rates less than 10^{-5}; hence, the threshold question has been avoided in this evaluation.

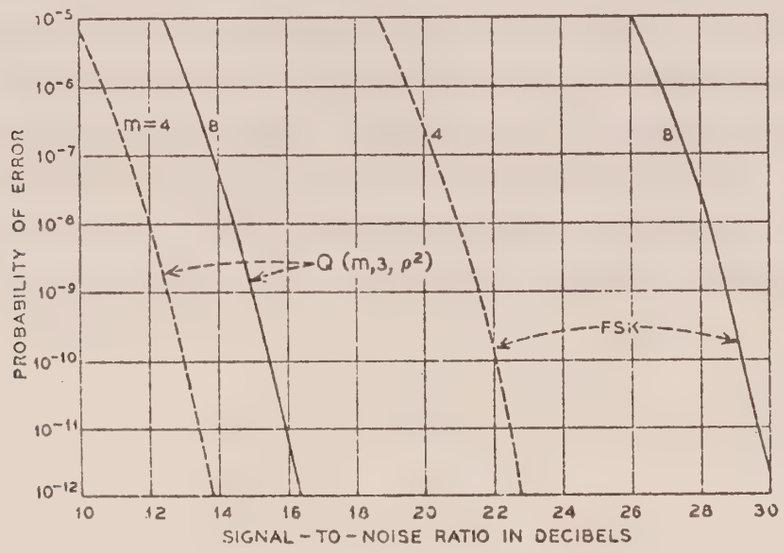


Figure 4-4. Comparison of the FSK System With the Optimal System, n = 3.

Though the performance of an FSK system appears to be substantially below that of the ideal receiver, this is not of much significance unless one can show that a receiver can practically be built that more closely approaches the limit of performance. Prabhu does this and shows that ideal performance for bandwidth expansion of 2 can be met using coherent phase shift keying and bandwidth expansion factor of 2. Therefore, it appears that the most effective modulation techniques for digital information are those which employ PSK modulation and coherent detection, but spread the bandwidth by transmitting redundant information in the form of codes. This, of course, increases the complexity but should provide much better performance than simpler systems that expand bandwidth such as wide-deviation fm. From a philosophical point of view, it may be worthwhile to emphasize the difference in the coding versus wide-deviation fm techniques. Fm is basically an analog technique. One parameter we are normally

concerned with is the accuracy of reproduction or fidelity of the recovered signal which, in turn, is controlled by both linearity and noise. Assuming we can design sufficiently linear systems to meet our objectives, then accuracy of reproduction of the signal is totally controlled by noise, which we can decrease to arbitrarily low amounts by increasing C/N and/or deviation. Fm systems, therefore, have proven very effective where fidelity of the signal is important; for example, in the relay of multichannel, ssb channelized, telephony, and color television.

For digital transmission the situation is quite different in that we are primarily concerned with error probabilities of a binary signal. Further, moderate processing times are generally allowed, hence, one would almost intuitively conclude that the most efficient techniques for transmission of digital data would involve transmission of redundant information in the form of codes and/or increase the processing time beyond the period of one date symbol. This statement appears to agree well with current developments in digital transmission technology.

5.0 PHASE SHIFT KEYED SYSTEMS

The error rate characteristics of a typical 4-phase PSK system employing coherent detection are shown in figure 5-1. It will be noted that in the region of interest the error rate increases approximately one order of magnitude for each dB decrease in S/N. This agrees very well with the peak distribution of Gaussian noise as is shown by figure 5-2.

For some systems where interference limits error rate, a quite different situation exists. If the interferor is a constant envelope signal such as a CW carrier or fm-modulated signal, the signal-to-interference ratio may be much lower. For this situation, the allowed S/I is a function of the S/N since Gaussian (thermal) noise adds to the constant envelope interferor. This has been investigated in detail by Prabhu,[8] and results from this reference are shown in figure 5-3. Measured S/I for a 21.5-Mbit/s PSK system is shown in figure 5-4 for two values of S/N. Data in figure 5-4 were obtained by increasing interference of a CW signal until error rate was reduced to 10^{-6} and the S/I at this point recorded. This was done for various frequency offsets and it will be noted that position of interfering carrier in the band had little effect on error rate. The somewhat higher interference allowed at large frequency offsets is assumed to result from the bandpass filter in the modem. These tests were made by inserting the interfering signal at limiter input prior to demodulation and did not include the microwave if and preselector filtering which would provide further out-of-band rejection. The fact that the allowed interference level was relatively independent of frequency error is as would be expected from a simple model, illustrated in figure 5-5. The interfering signal phase modulates the desired signal in the manner shown. For large in-band offsets the peak phase error occurs more often but remains in peak phase error position for shorter periods of time; hence, the lower frequency offsets should cause increased bunching of errors but no change in error rate. Tests were made using a CW interferor but similar results were obtained using other constant envelope signals such as an fm signal.

It will be noted that the amount of interference that could be tolerated by this modem increased slightly when the CW interferor was near the center frequency of the channel. The reason for this is not clear, though it appears likely that the effect is related to the behavior of the carrier and/or clock recover circuits.

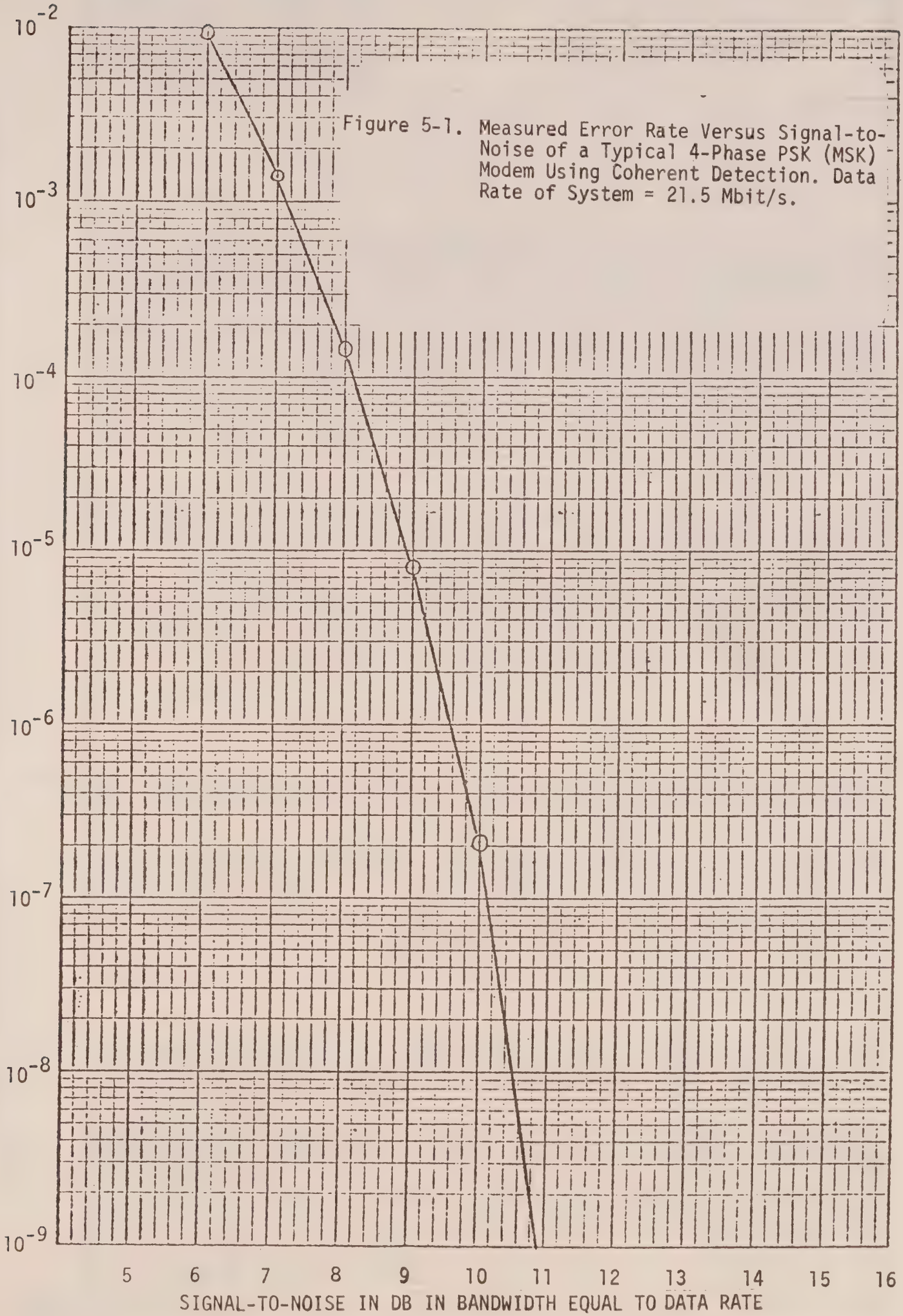


Figure 5-1. Measured Error Rate Versus Signal-to-Noise of a Typical 4-Phase PSK (MSK) Modem Using Coherent Detection. Data Rate of System = 21.5 Mbit/s.

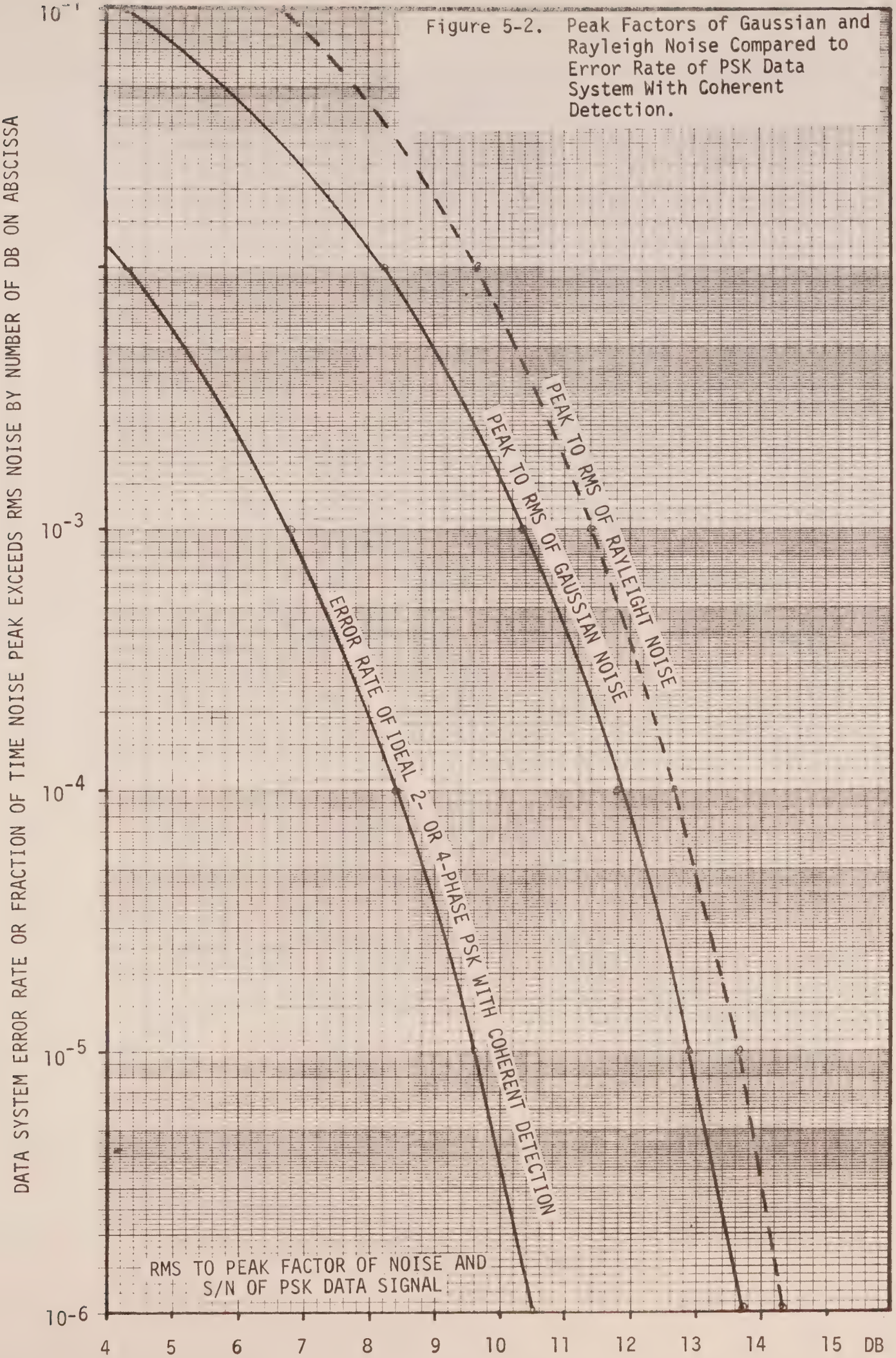
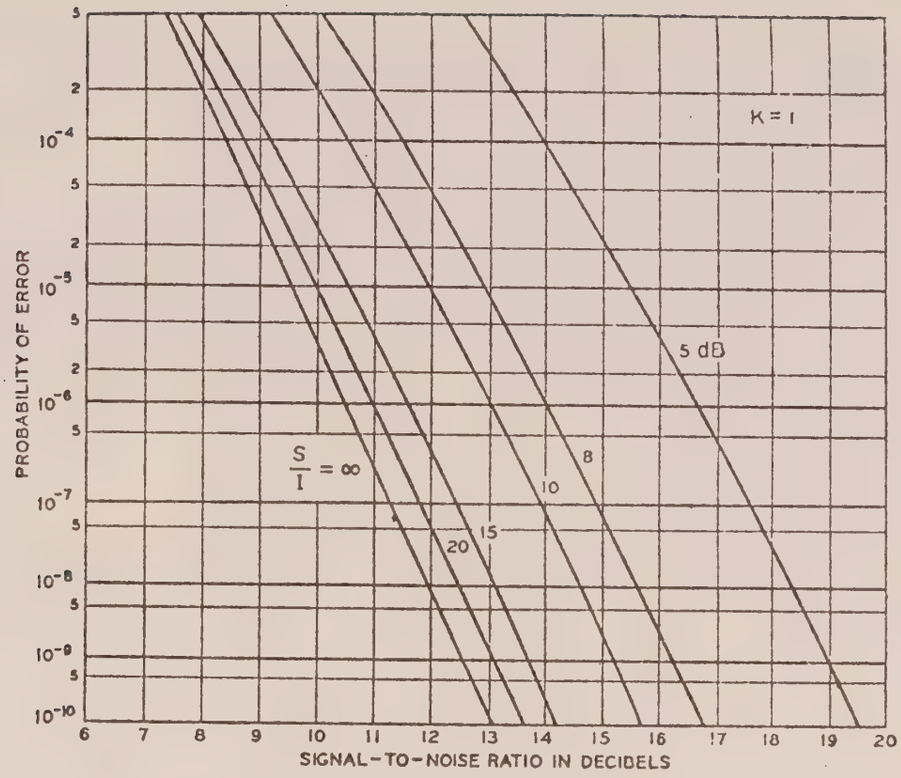
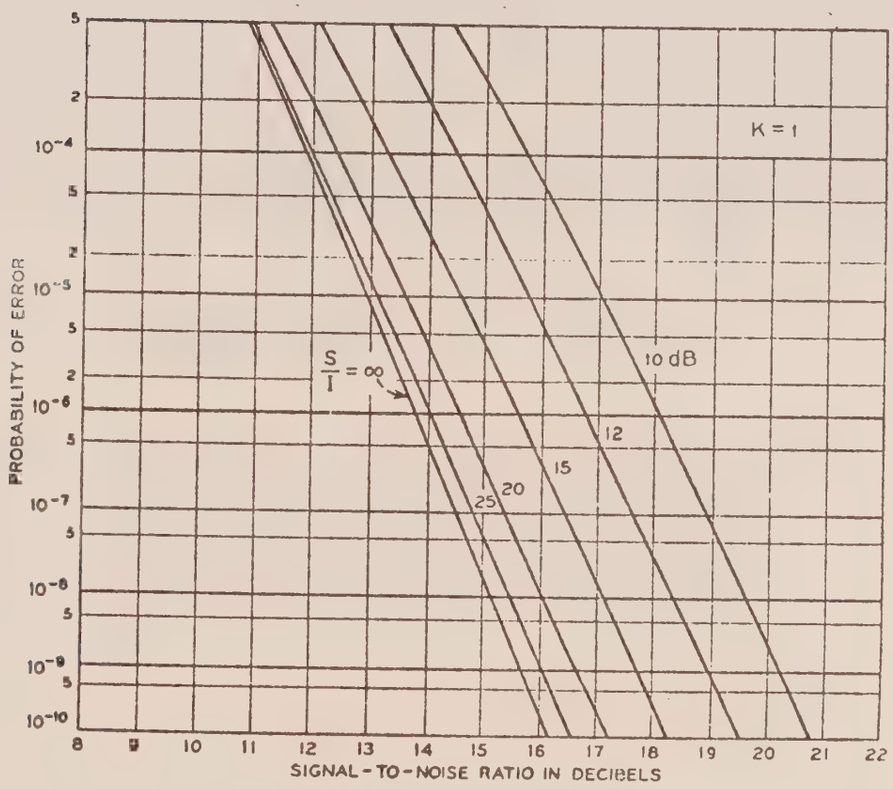


Figure 5-2. Peak Factors of Gaussian and Rayleigh Noise Compared to Error Rate of PSK Data System With Coherent Detection.



a. — Error rates for a 2-phase CPSK system with one interferer.



b. — Error rates for a 4-phase CPSK system with one interferer.

Figure 5-3. Error Rates for 2- and 4-Phase PSK Systems Employing Ideal Coherent Detection. (Data from Prabhu.)[8]

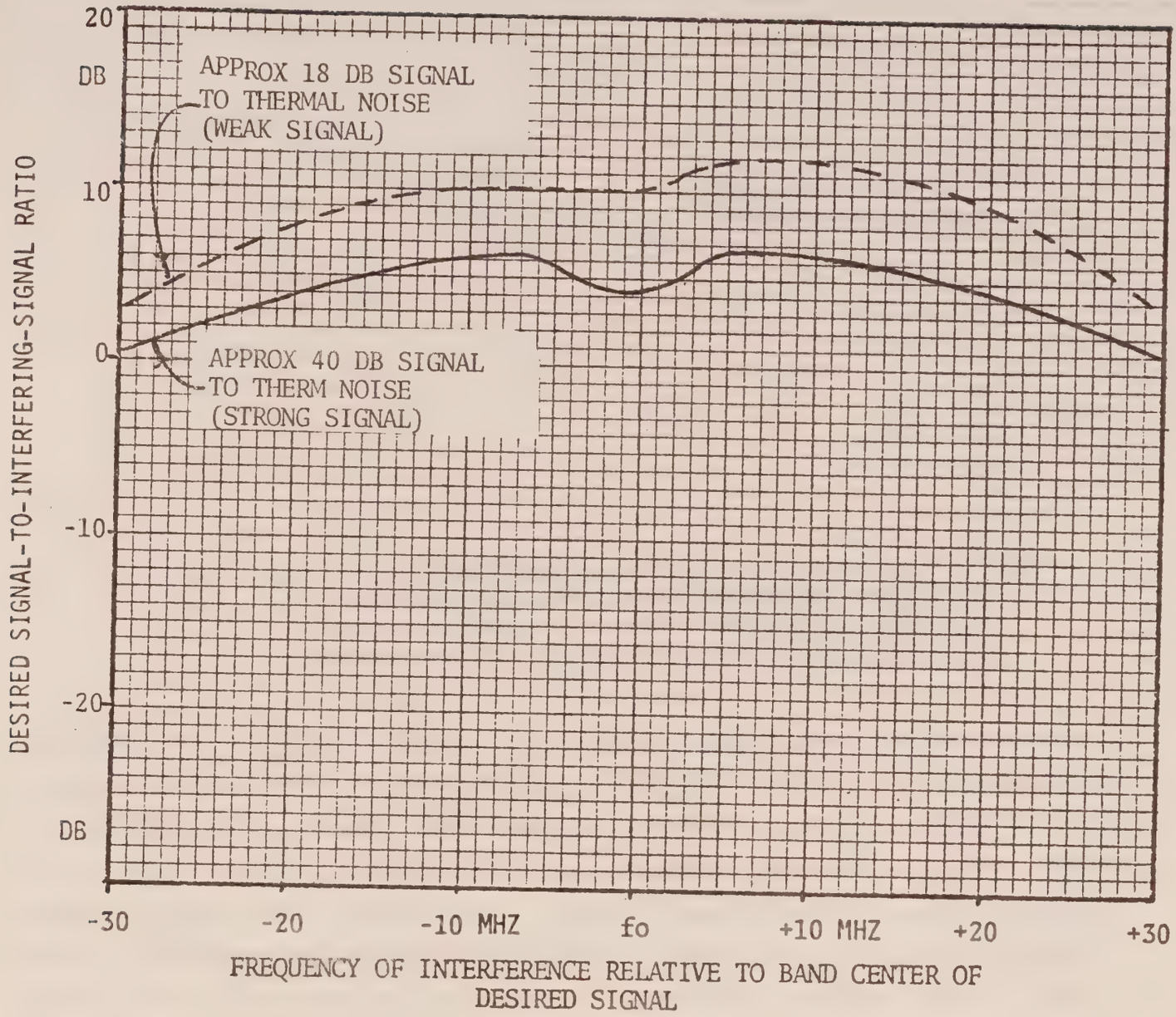


Figure 5-4. Measured Signal-to-Interference Ratio for an Error Rate of 10^{-6} as a Function of Frequency from Band Center of CW Intererror for a 21.5-Mbit/s, 4-Phase PSK (MSK) Data Modem Employing Coherent Detection. Interfering signal was inserted into limiter prior to data demodulator but following microwave receiver if filtering.

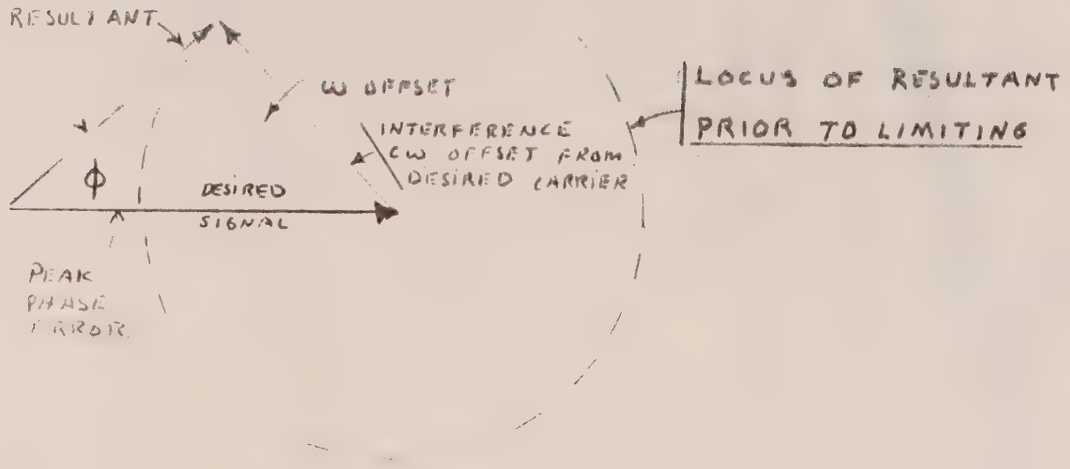
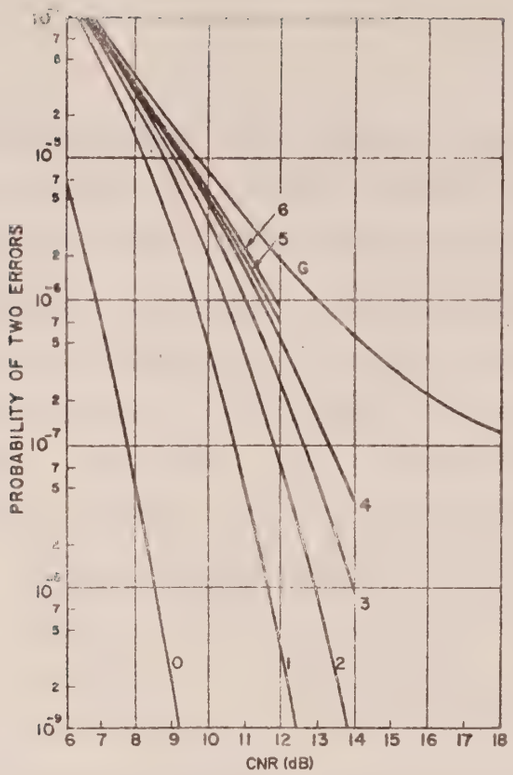
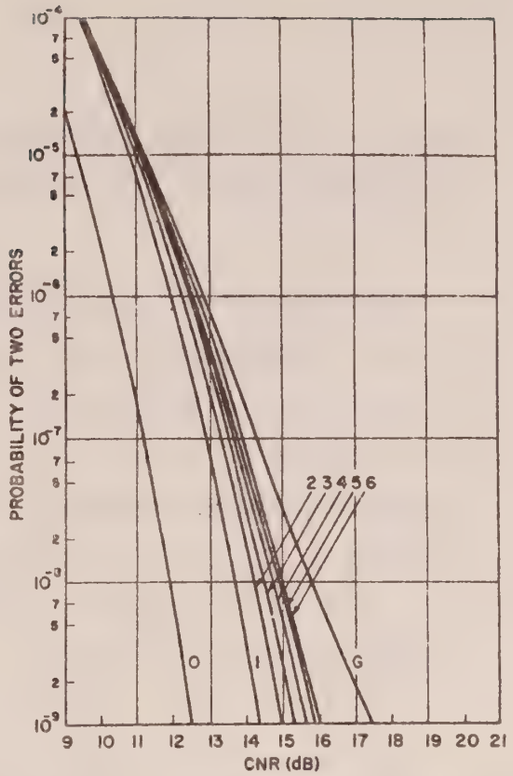


Figure 5-5. Vector Relationships of Desired Signal Carrier, CW Interferor With Frequency Offset and Resultant Prior to Limiter. Limiter removes am component and produces a signal with only phase modulation.

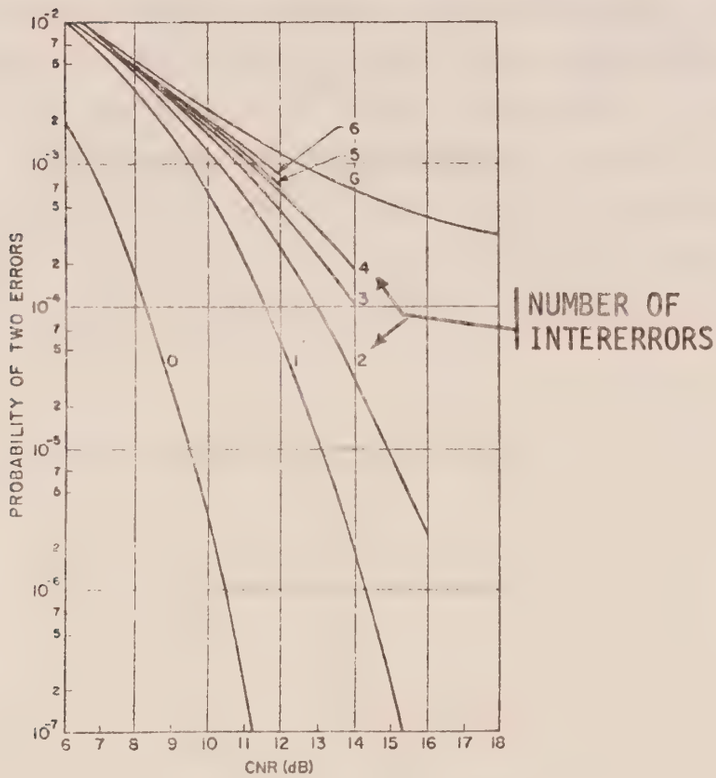
Goldman[9] has analyzed in detail the effect of interference on the error rate characteristics of PSK system and some of the information from this reference is included as figure 5-6. This analysis considers a varying number (1 through 6) of equal-level cosinusoid interferors and added Gaussian noise (ie, infinite number of equal-level interferors). These curves illustrate again the importance of considering the peak factor of the interference as the peak factor affects the error rate by 2 to 3 orders of magnitude at some values of CNR. Goldman states "The results indicate that one cannot approximate well the effect of interference on the performance of a PSK system by treating it as additional Gaussian noise." This might lead one to assume that the interference model (Gaussian noise) previously recommended is a poor one. However, it appears that Goldman's statement should only be applied to the case of one or two interferors. It will be noted from Goldman's data in figure 5-6 that for as small a number as six equal-level interferors, the error performance closely approaches that of an added equal-level of Gaussian noise. Therefore, the recommended model appears to be a good one, provided one recognizes that it is a worst-case model and that on an average error rate, as a function of S/I, will be somewhat better than predicted.



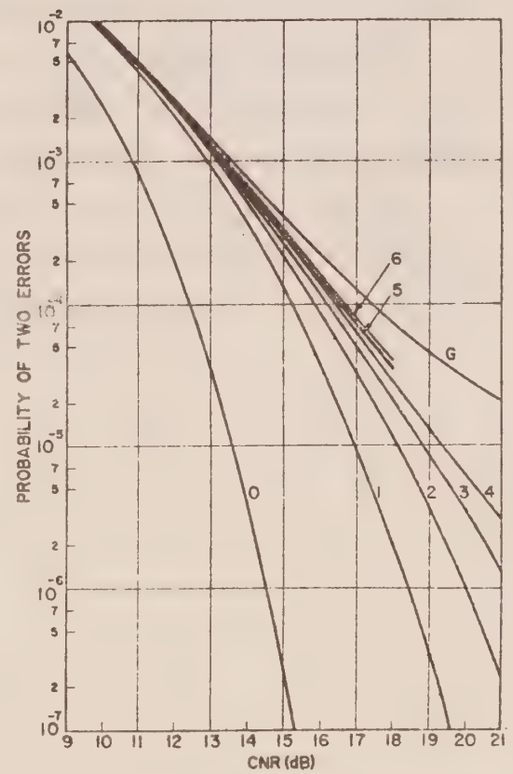
a. Probability of two errors versus CNR for CPSK, 2-phase, CIR = 8.



b. Probability of two errors versus CNR for CPSK, 4-phase, CIR = 14.



c. Probability of two errors versus CNR for DPSK, 2-phase, CIR = 8.



d. Probability of two errors versus CNR for DPSK, 4-phase, CIR = 14.

Figure 5-6. Probability of Two Consecutive Errors as a Function of Carrier-to-Noise Ratio (CNR) for Carrier-to-Interference Ratios of 8 and 14 dB and for 0 to 6 Cosinusoid Intererrors.

(From Goldman.)

It should be noted that Goldman's error rate curves are for probability of two consecutive errors, though he states that "For CPSK the conditional probability of error equals the single-error probability."

These curves by Goldman illustrate another characteristic worth emphasizing. That is the difference in the performance of coherent and differentially coherent detection. For example, for a 2-phase PSK system, 12-dB S/N and one interferor, the error rate is about 3×10^{-9} and 5×10^{-5} (about 4 orders of magnitude different) for coherent and differentially coherent detection respectively. Though the differentially coherent error curve is not plotted to an error rate of 5×10^{-9}, an extrapolation of this curve indicates that S/N would have to be increased by about 5 dB (ie, to 17 dB S/N) to make the differentially coherent system perform at the same error rate as the coherent detection system operating at 12 dB S/N. This is important for the system being considered since differentially coherent detection is simple, and, hence, economical. However, the interference levels (-8 and -14 dB) for which these curves were plotted, exagerate this difference, and a reduction of S/I by 2 dB is likely to substantially reduce this differential. In Goldman's examples he was most interested in showing the effect of the number of interferors (which is equivalent to specifying peak factor) on the error rate at fixed S/I and this is most obvious at low signal-to-interference ratios chosen for the examples. Therefore, these data should not rule out consideration of differentially coherent detection for the type system being considered.

6.0 SUMMARY AND CONCLUSIONS

Since the system being considered is expected to operate in a high interference environment, the definition of the interference model is the key to defining the signal-to-interference ratios that can be tolerated by either fm or PSK systems. It appears reasonable to model the interference as Gaussian noise since multiple interferors are expected. The model, at least, appears to be a safe one.

It does not appear that wide deviation ratio fm (FSK) offers the "capture ratio" advantage expected for the digital fm , at least, for the assumed interference model because of the poorer threshold characteristics. However, error rate performance of wide- and narrow-deviation systems is not compared in these notes, and it is pointed out that error rate performance cannot be derived directly from baseband noise near threshold because of the very non-Gaussian distribution of the noise. The "click theory" of threshold noise is introduced in a somewhat general fashion, and further investigation appears desirable to numerically define expected error rate performance.

Although PSK modulation techniques have not been discussed in detail in these notes, reference is made to work by others that indicates that coherent detected PSK performance approaches the theoretical ideal performance derived from purely information-theory considerations. The problems and limitations in the application of PSK techniques appear to be a desirable subject for future notes since application of coherent detection processes presents implementation difficulties when operated in the burst communication mode. Use of differentially coherent detection appears to offer a simple solution to this problem and also appears to offer a good compromise between cost and performance.

REFERENCES

[1]Hubbard, W.M., "The Efect of Intersymbol Interference on Error Rate in Binary Differentially Coherent Phase-Shift-Keyed Systems," BSTJ, July-August 1967, pp 1149-1172.

[2]Harris, D. P., "Noise, Interference, and Cross-Modulation on High-Density FM Communications Systems," M.S. Thesis, Southern Methodist University, June 1958.

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[5]Bennett, W.R., and Rice, S.O., "Spectral Density and Autocorrelation Functions With Binary Frequency-Shift Keying," BSTJ, September 1963, pp 2355-2385.

[6]Rice, S.O., Noise in FM Receivers, M. Rosenblah, Editor, "Time Series Analysis," John Wiley and Sons, Inc., New York, 1963.

[7]Cohn, J., "A New Approach to the Analysis of FM Threshold Detection," Proc. NEC, Vol 12, pp 221-236, 1956.

[8]Prabhu, V.K., "Error Rate Considerations for Coherent Phase-Shift Keyed Systems With Co-Channel Interference," BSTJ, March 1969, pp 743-767

[9]Goldman, N., "Multiple Error Performance of PSK Systems With Co-Channel Interference and Noise," IEEE Transactions on Communications Technology, Vol. COM-19, No. 4, August 1971; pp 420-430.

APPENDIX I

I. INTRODUCTION

A technique of deriving the S/N and threshold characteristic of a FM receiver is described below. This technique was derived by D. P. Harris[1] and has proven very convenient and reliable in defining the performance of FM receivers using discriminator detection. Since defining the strong signal characteristics of FM receivers is a well known and straightforward process, the value of the technique lies in defining the threshold performance of FM receivers. Since many design faults in FM receivers, such as poor limiting performance and incorrect limiter time constants, become more important in the threshold region, comparison of a particular receiver design to threshold performance predicted by this technique has been a valuable aid to optimization of equipment design. Theoretical proof of technique is not included in this appendix but characteristics predicted by this technique agree closely with measured performance of well designed FM receivers.

2. DESCRIPTION OF TECHNIQUE

2.1 General

Usually we are concerned with not only the baseband S/N of an FM receiver but also the noise spectral density over the baseband. This is of particular interest when the baseband is channalized as, for example, in microwave relay of multichannel telephony, but even in single channel systems such as FM broadcasting and mobile radio it is common practice to employ signal pre-emphasis prior to modulation with matching de-emphasis in the receiver to optimize the S/N. For proper design of emphasis, both the baseband signal spectral density and receiver noise spectral density are important considerations.

If the FM signal exceeds the total noise power in the receiver bandwidth by at least 12 db, the S/N in a defined segment of the baseband is given by:

$$S/N\Big|_{cH} = \left(\frac{C}{N}\right) 2BW_{ch} \left(\frac{\Delta f}{f_{ch}}\right)^2 \qquad (1)$$

$$S/N\Big|_{cH} = \left(\frac{C}{N}\right)_{BW,if} \left(\frac{BWif}{2BW_{ch}}\right) \left(\frac{\Delta f}{fch}\right)^2 \qquad (2)$$

Where:

$S/N\Big|_{ch}$ is S/N in the Defined Channel

$\left(\frac{C}{N}\right)$ = Carrier to noise in the Receiver (if) bandwidth or in twice bandwidth of channel as designated by subscript of "if" or $2BW_{ch}$

Δf = peak deviation

fch = baseband frequency of the channel

It is seen that for the strong signal condition the channel S/N is independent of the receiver bandwidth assuming, of course, that the bandwidth is at least sufficient to pass the signal. The minimum bandwidth (Carlsons Rule) is defined as

$$BW_{min} = 2(f_{ch\ max} + \Delta f).$$

The receiver BW does effect the threshold characteristics and Cohn[2] has shown that this is due to noise impulses of an amplitude and phase sufficient to cause addition or subtration of one complete rotation of the carrier vector. Harris has taken the relations for the spectral density of the inpulsive noise so produced and equated it to the noise relation shown by equation one and solved the result in terms of C/N and ratio of channel frequency to receiver bandwidth. The result is the curve given in Figure I-1. This curve then defines for any ratio of channel frequency to receiver bandwidth the $C/N\Big|_{BWif}$ at which the inpulsive noise is equal to the random noise defined by equation 1. At the C/N point so defined the channel S/N is 3 db less than that defined by equation 1. This is somewhat arbitrarily defined as threshold since it is a convenient reference. The main interest in the curve shown in Figure I-1 is that it leads to the simple technique for constructing receiver S/N curves down through the threshold region as described below.

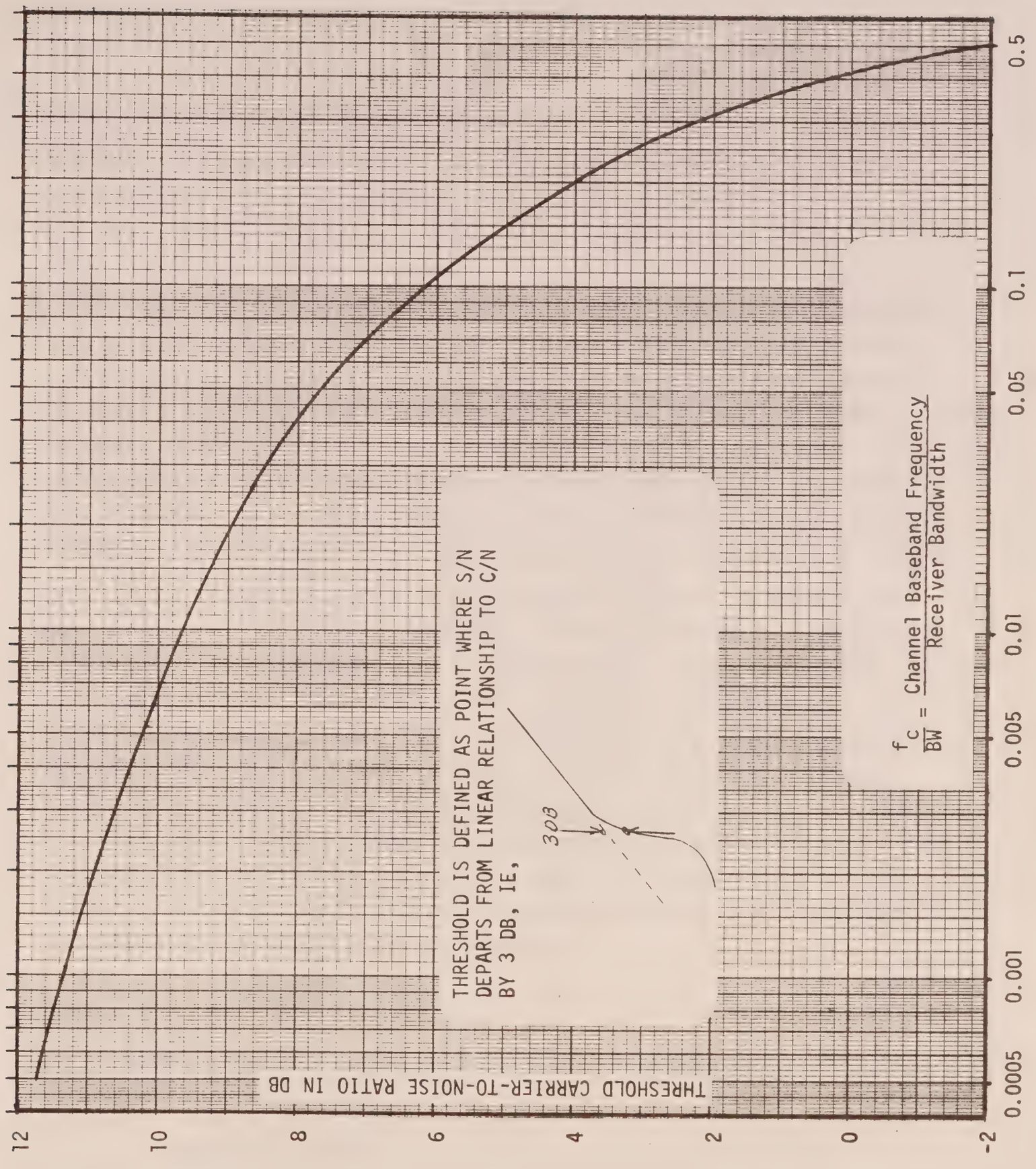


Figure I-1. FM Channel Threshold as a Function of Ratio of Baseband Frequency to Receiver Bandwidth.

2.2 Plotting of Receiver S/N Characteristics

The family of curves which define the S/N over the baseband may be rapidly constructed in the following manner. Chose a set of channel frequencies spaced by about one octave and a C/N known to be well above threshold (e.g., 30 db C/N). Choose a convenient channel bandwidth which is small relative to the total baseband bandwidth of interest. In a channelized system this will be defined by the channeling, or chose a convenient bandwidth such as 1 Hz or 1 kHz which will allow easy conversion to noise spectral density. If curves are to be plotted for one octave steps over the baseband, then it is only necessary to make one calculation, say for the highest frequency and step of the S/N in 6 db steps for each octave. Through the points so established draw a straight line with S/N increasing linearly with C/N. For each channel frequency chosen determine ratio $\frac{f_{ch}}{BW_{if}}$ and for each ratio tabulate C/N obtained from curve in Figure I-1. This, then, is the C/N at which the channel S/N departs 3 db from the linear relationship. Place this point for each channel on the graph. As the signal fades into the threshold region the baseband noise spectral density approaches a flat distribution. Therefore, the S/N for the low frequency channels approaches that of the high frequency channel as the system is faded into the threshold region and the S/N of the lowest channel will pass through all the 3 db threshold points of all higher channels. Therefore, draw a curve through these points and smoothly bend these into the straight line portion of the curves. The result is a family of curves that accurately define the receiver baseband noise distribution from strong signal condition well down into the threshold region.

2.3 Example

The following parameters are chosen as an example and the result is presented in Figure I-2.

$f_{ch(Max)} = 100$ kHz

$\Delta F = 1$ mHz peak

$BW_{ch} = 1$ kHz

$$BW_{if} = 2(f_{ch(max)} + \Delta f) = 2(0.1 + 1.0) = 2.2 \text{ mHz}$$

at 30 db C/N and 100 kHz (top of baseband) S/N is computed as:

$$S/N\Big|_{ch} = \left(\frac{C}{N}\right)_{if} + 10 \log \left(\frac{BW_{if}}{2BW_{ch}}\right) + 20 \log\left(\frac{\Delta f}{f_{ch}}\right)$$

$$S/N\Big|_{ch} = 20 + 10 \log \frac{2.2}{2(.001)} + 20 \log \left(\frac{1}{0.1}\right)$$

$$S/N\Big|_{ch} = 30 + 30.4 + 20 = 80.4 \text{ db}$$

For lower channels S/N increases 6/octave or 20 db/decode. Therefore, for 30 db C/N the S/N of various channels to be plotted are:

Ch Freq.	S/N	f_{ch}/BW	C/N at 3 db break
100 kHz	80.4	.045	7.85 db
50 KHz	86.4 db	.023	8.8
25 KHz	92.4 db	.015	9.25
10 KHz	100.4 db	.0045	10.3
5 KHz	106.4	.0023	10.8

f_{ch}/BW is tabulated for these channel frequencies and C/N at 3 db break points from Figure I-1 are tabulated. These points are then plotted and curves drawn as shown in Figure I-2.

REFERENCES:

[1] Harris, D.P., "Noise, Interference, and Cross-modulation on High-Density FM Communications Systems", M.S. Thesis, Southern Methodist University, June 1958.

[2] Cohn, J., "A New Approach to the Analysis of FM Threshold Detection", PROC. NEC Vol. 12, pp 221-236, 1956.

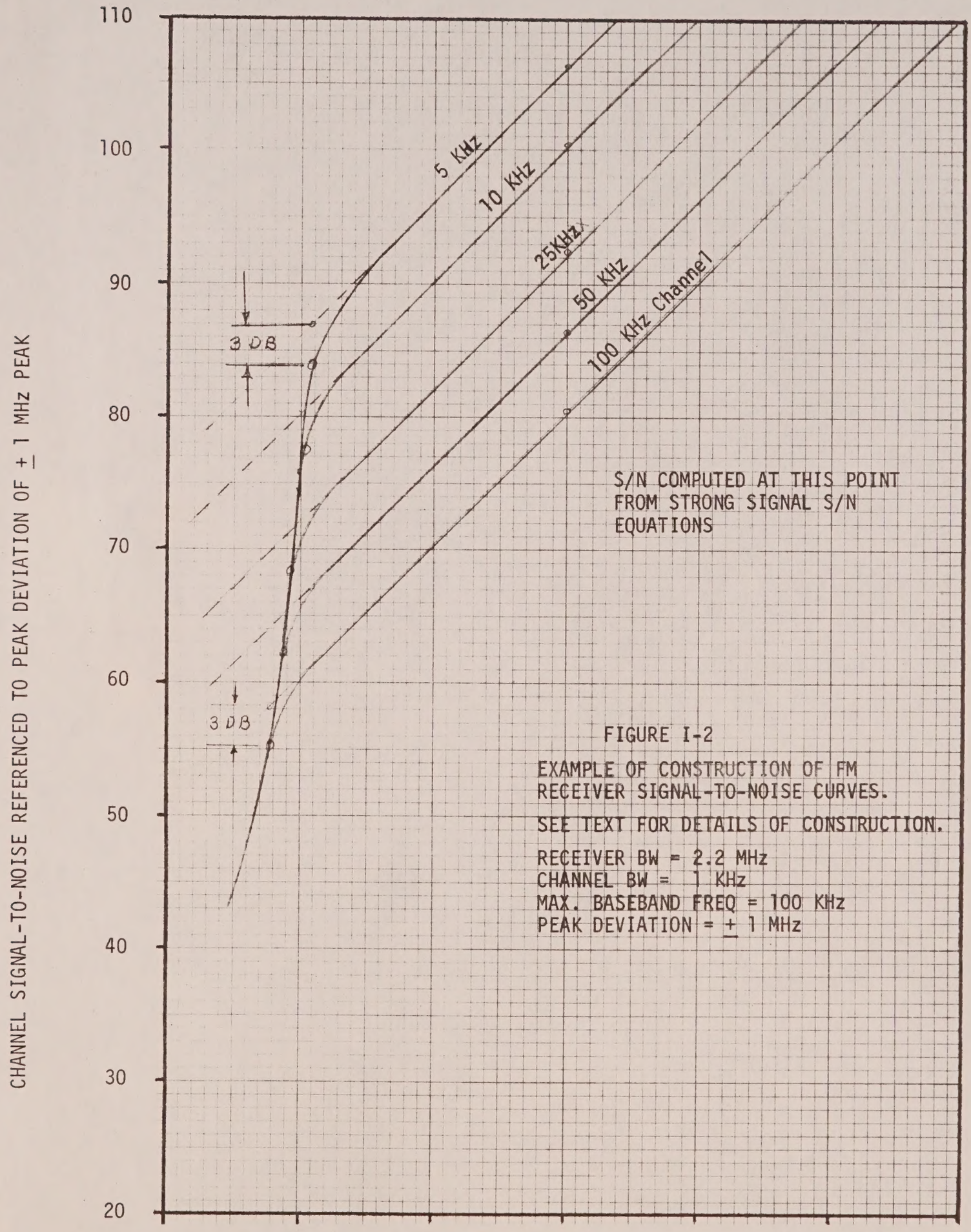
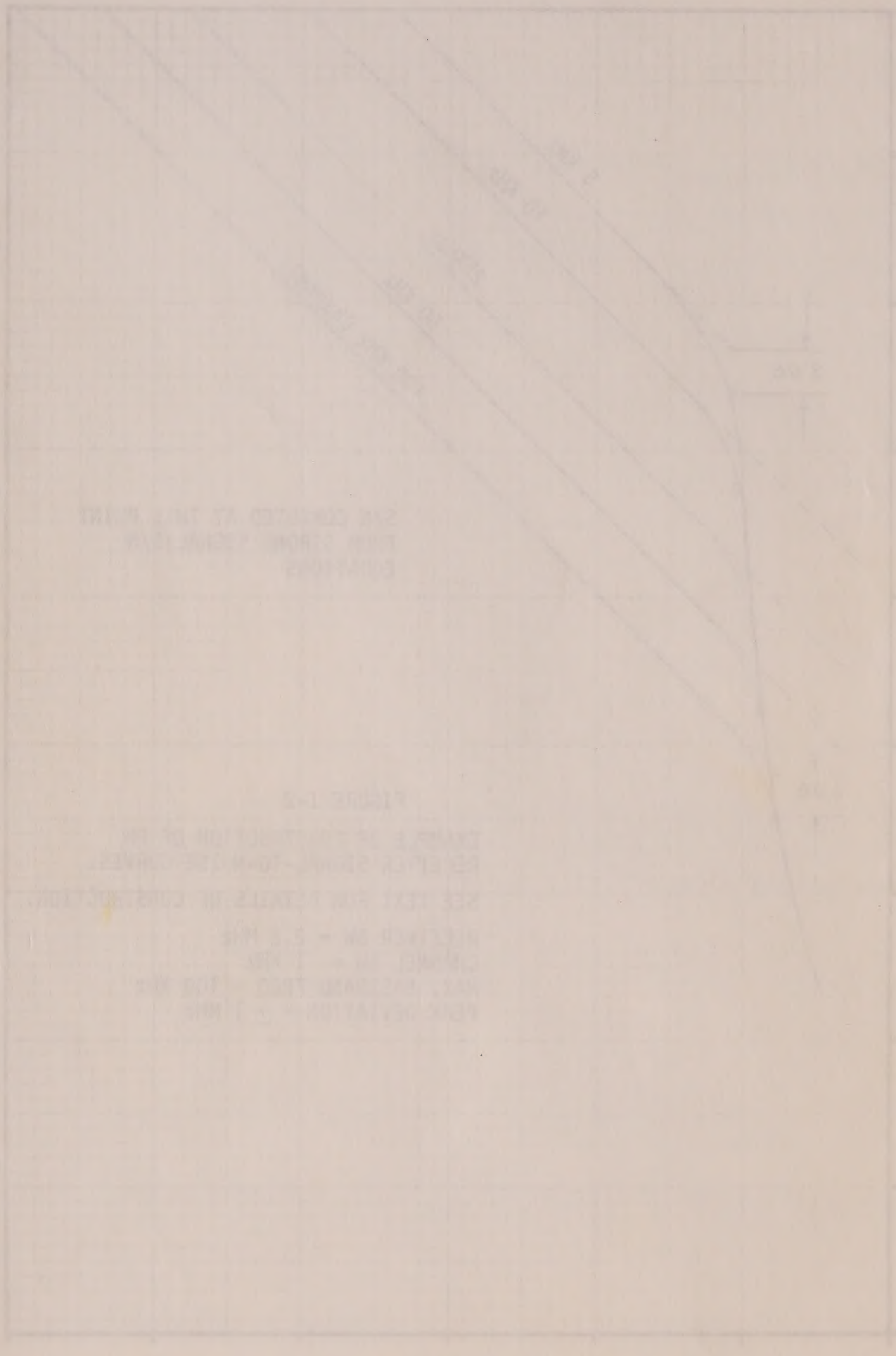


FIGURE I-2

EXAMPLE OF CONSTRUCTION OF FM RECEIVER SIGNAL-TO-NOISE CURVES.

SEE TEXT FOR DETAILS OF CONSTRUCTION.

RECEIVER BW = 2.2 MHz
CHANNEL BW = 1 KHz
MAX. BASEBAND FREQ = 100 KHz
PEAK DEVIATION = ± 1 MHz